Use of CAD software to solve trigonometry and vector problems

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Abstract

The engineering department at Indiana University-Purdue University Fort Wayne began offering a newly designed first-year curriculum in the fall 2014 semester with the intent to expose students to important mathematical techniques through engineering applications and to develop the students' problem solving abilities. The two new first-year courses are divided into three components: lecture, project studio, and computer lab, which are designed to complement one another. Throughout the courses, students are exposed to and required to use multiple solution procedures to solve problems. Moreover, students are required to document and communicate solutions in multiple representations to demonstrate a deeper understanding. The purpose of this paper is to explain how CAD (computer-aided drawing) is used by students to first solve and then document trigonometry and vector problems.

Introduction

As a result of its assessment-based, continuous improvement process, the engineering department at Indiana University-Purdue University Fort Wayne began offering a newly designed first-year curriculum in the fall 2014 semester. The overarching motivation behind the curriculum change was the desire to expose students to important mathematical techniques through engineering applications and to develop the students' problem solving abilities. The curriculum change involved replacing four courses with two courses, as shown in Table 1.

Table 1: Comparison of old and new first-year engineering curriculum

old curriculum			new curriculum		
number	title	credit hours	number	title	credit hours
ENGR 101	Introduction to Engineering	1	ENGR 127	Fundamentals of Engineering I	4
ENGR 120	Graphical Communication and Spatial Analysis	2			
ENGR 121	Computer Tools for Engineers	2	ENGR 128	Fundamentals of Engineering II	4
ENGR 199	Introduction to Engineering Design	3			

The textbook for the courses is *Introductory Mathematics for Engineering Applications by* Rattan and Klingbeil.[1] Specifically, the first course in the new sequence, ENGR 127, seeks to prepare students for the study of engineering through learning how to:

1. effectively approach the study of engineering,
2. rigorously apply of mathematical techniques in engineering particularly algebra, trigonometry, descriptive statistics, & simple derivatives
3. carry out a disciplined engineering project,
4. prepare and use graphical objects (graphs, tables, drawings, charts) for technical communication, and
5. use spreadsheet and CAD software.

Both courses in the new curriculum contain three components: a lecture, project studio, and laboratory component. In ENGR 127, the laboratory component introduces engineering computer tools for computer-aided drawing (CAD) and visualization, as well as spreadsheet calculation. The CAD software used in the course is AutoCAD.

The three components of the course are designed to complement one another, i.e. students often work in the computer lab and studio on problems that have been introduced in lecture. As much as possible, class time makes use of active learning exercises—this is especially true in the project studio and computer lab.

Throughout the course, students are exposed to and required to use multiple solution procedures to solve problems. Moreover, students are required to document and communicate solutions using multiple representations to demonstrate a deeper understanding of the material.

Teaching multiple solution methods and requiring multiple representations to communicate and document solutions is supported by educational research. For example, two overarching implications of Gardner's Multiple Intelligences Theory[2] are *Individualization* and *Pluralization.* Individualization refers to the fact that no two students think alike and educators should take that in to account as much as possible when teaching and evaluating. Pluralization refers to the idea that important material should be taught in several ways. This is important for two reasons. First, different students learn differently and a method that works for one student might not work with another—using multiple methods has a greater potential to reach more students. Second, teaching several different methods demonstrates a deep understanding by the instructor, and students who master different methods demonstrate a deeper understanding of the topic.[2]

In teaching sciences, DeMao "claims that providing students with a choice of methods as well as requiring additional methods as a way to validate results can be beneficial to student learning…It is argued that conceptual understanding can be enhanced through the use of multiple methods in an environment where students can compare, evaluate, and verbally discuss competing methodologies through the facilitation of the instructor."[3]

Similar observations were made in the study of engineering problem solving, especially as applied to ill-structured or work-place engineering problems. Jonassen, Stobel and Lee, [4] in compiling a case library of engineering problem solving, noted that:

- Ill-structured problems are solved in many different ways. Engineers should be able to identify and evaluate different solution methods. [4]
- Expert engineers are able use multiple ways to represent problems. Multiple representations of problems helps to deepen understanding and clarify aspects of the problem. It also can stimulate alternate solution techniques. [4]

To see how multiple solution techniques and multiple representations are stressed in this course, consider the two-link planar robot, which is a reoccurring problem in the course text.[1] Solution methods based on trigonometric equations are introduced in chapter 3 and solved by students with pencil, paper, and calculator. Students are then introduced to a vector approach in chapter 4, and in chapter 5 complex numbers are applied to the same problem. In the computer lab, student use spreadsheets to perform some of the calculations required with these methods. Also, in the computer lab students use a CAD package to solve the same problem. The CAD representation is also used to document and communicate the solution to the problem.
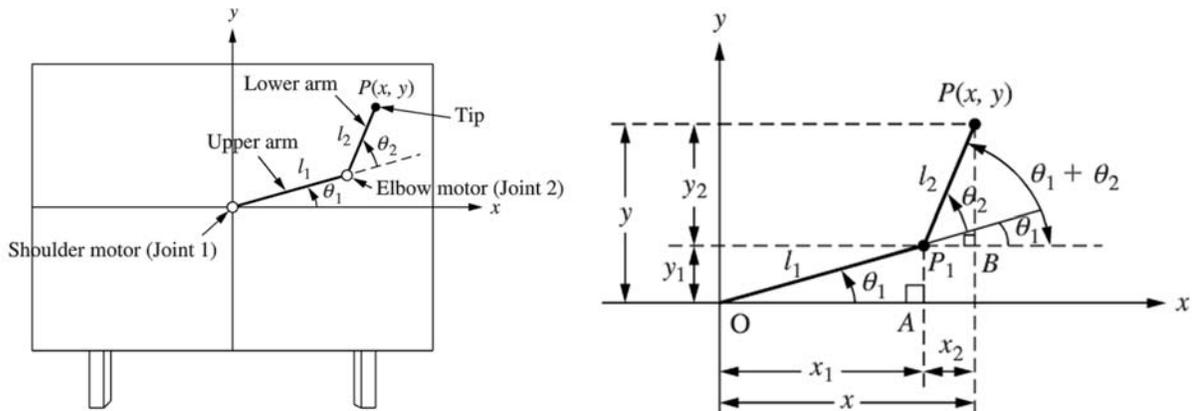

Figure 1: Schematic representation of a two-link planar robot.[1]

The purpose of this paper is to describe how a CAD software package, in this case AutoCAD, has been used to solve trigonometry and vector problems. This paper presents the general CAD concepts, as well as the specific AutoCAD commands, that are necessary to solve simple trigonometry and vector problems. Several illustrative examples are presented, and preliminary student feedback is discussed.

CAD Background

Students in the class are assumed to no prior experience with CAD. Therefore, a brief introduction to the software, including instruction on selection techniques, coordinate entry, and basic commands is required. A list of the commands required to work the examples provided in this paper is given in Table 2.

Table 2: List of AutoCAD menus/commands needed to solve example problems 1-5

Draw	Modify	Utilities
Line	Erase	Measure
Circle	Trim	Area
Polyline	Move	
Drafting settings	Annotation	Coordinate entry
Grid	Single Line text	Absolute Cartesian
Snap	Multiline text	Relative Cartesian
Osnap	Multileader	Relative polar
(endpoint/intersection)	Linear Dimension	
Polar tracking	Align Dimension	
Ortho	Angle Dimension	

As in a traditional CAD class, students are given a few simple drawings, both as in-class exercises and homework to practice basic commands and simple drawing techniques. Also students are provided two drawing templates, based on the acad.dwt template and acadiso.dwt template, that they are advised to use to begin drawings. The templates have several different layers as shown in Figure 2 with lines of different color, type, and weight. A border and titleblock are place on layer 0 in the paper space on layout 1. Also, the dimensioning styles are modified.

Name	On	Freeze	Lock	Color	Linetype	Lineweight
0	☀	☀	🔓	□ white	Continuous	▬ 0.80 mm
CENTER	☀	☀	🔓	■ green	CENTER	— 0.30 mm
CONS	☀	☀	🔓	□ yellow	Continuous	— Default
Defpoints	☀	☀	🔓	□ white	Continuous	— Default
DIMEN	☀	☀	🔓	■ magenta	Continuous	— Default
HIDDEN	☀	☀	🔓	■ red	HIDDEN	— 0.30 mm
TEXT	☀	☀	🔓	■ blue	Continuous	— Default
VISIBLE	☀	☀	🔓	□ white	Continuous	▬ 0.60 mm

Figure 2. Defined layers in the student-provided templates.

Presentation of Example Problems

In the next section, five typical example problems are presented—the first four problems are from Ref. [1] and the last problem is a statics problem [6]. The problems were assigned either as in-class exercises or homework. Solutions are provided to illustrate the procedure. Some solutions are not complete to emphasis the steps involved in the process.

Example Problem 1

3-2. The eyes of a 7 ft 4 in. player are 82 in. from the floor, as shown in Fig. P3.2. If the height of the basketball hoop is 10 ft from the floor, find the distance l and angle θ from the player's eye to the hoop.
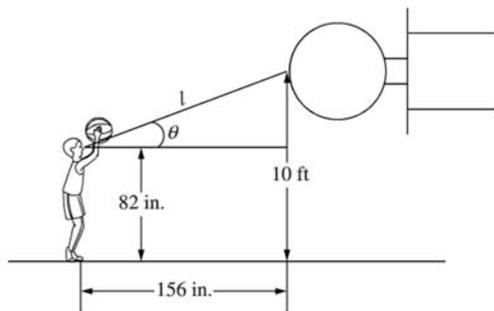
Figure P3.2 A basketball player in front of the basketball hoop.
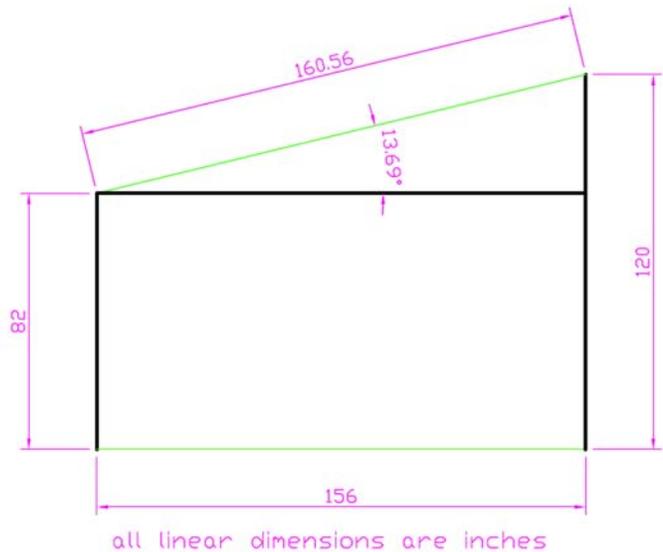
Figure 3. Example problem 1 statement[1] and solution.

Example problem 1 is a straightforward trigonometry problem. The given geometry is constructed using snap-to-grid to locate a start point and then relative Cartesian or polar coordinate entry to complete the drawing. The construction line (green) from the player to hoop is drawn using the line command. The points are located with Osnap-endpoint. The distance is indicated using dimensioning-align and angle is found using dimensioning-angle.

Example Problem 2

Example problem 2 (see Figure 4) is a geometry problem. The given geometry is constructed using snap-to-grid to locate a start point and then relative Cartesian or polar coordinate entry to complete the drawing. The construction line (green) is drawn from the top corner, starting with Osnap-endpoint using relative polar coordinate, i.e. @250<-120. The area is computed using Utilities-Area. To finish the drawing, the Trim command could be used to cut the construction line, and the line could be transferred to the visible layer to appear as a boundary.

To report the solution, text was added using Multiline text with font changed to Verdana and superscript added with the code \U+00B2.

3-5. The same coop student calculates the area of lot 2 shown in Fig. P3.5 as 50,000 m². Is this the correct answer? If not, find the correct answer.
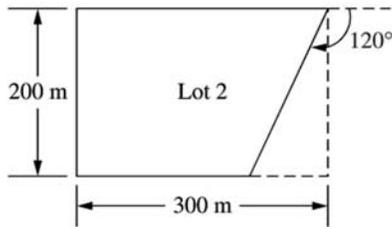

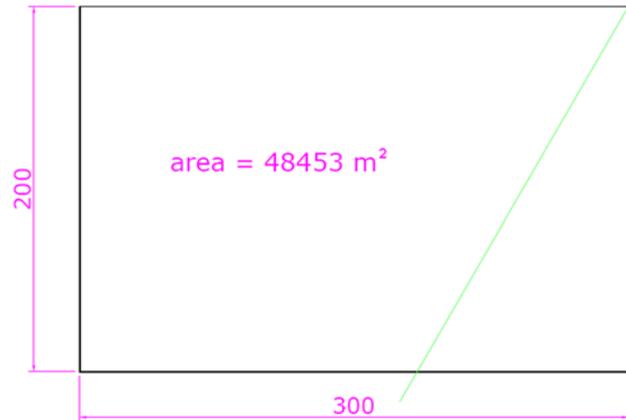
Figure P3.5 Dimension of lot 2 in the new subdivision.

Figure 4. Example problem 2 statement[1] and solution.

Example Problem 3

3-7. A truss structure consists of three isosceles triangles as shown in Fig. P3.7. Determine the angle θ using the laws of cosines or sines.
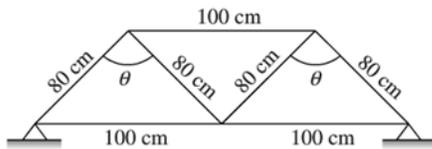
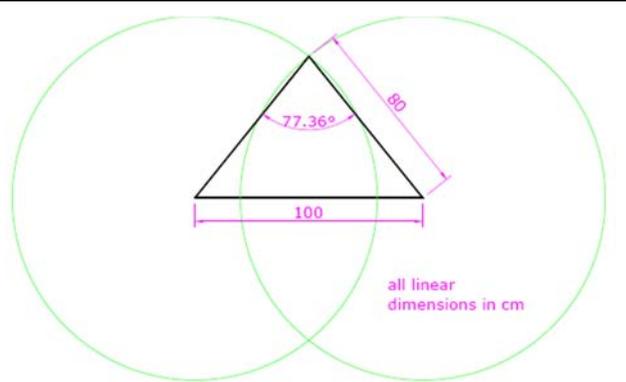**Figure P3.7** Truss structure for problem P3-7.

Figure 5. Example problem 3 statement[1] and solution.

Example problem 3 is more challenging as geometric techniques must be used to complete the solution. Often in problems where no angles are given, construction lines or construction circles are helpful to complete the problem. To begin, snap-to-grid is used to locate a start point. A horizontal line of 100 units is drawn. The two circles with a radius of 80 units are constructed using Osnap-endpoint of the horizontal line as the center. Finally, the angle is measured with dimensioning-angle.

Example Problem 4

3-46. In a motion capture study of a runner, one frame shows the subject supporting her weight on one leg, as shown in Fig. P3.46. The length of the foot segment (from ankle to toe) is 8 in. and the length of the lower leg (from ankle to knee) is 18 in.

(a) Given the angles shown in Fig. P3.46(a), find the position of the knee if the runner's toes touch the ground at the point $x = y = 0$.

(b) Now suppose that the same leg is positioned such that the knee is located in the second quadrant and oriented in the knee-down position (clockwise direction), as shown in Fig. P3.46. If the toes are located at $P(x, y) = (-6.25$ in., 25 in.), determine the values of θ_1 and θ_2.
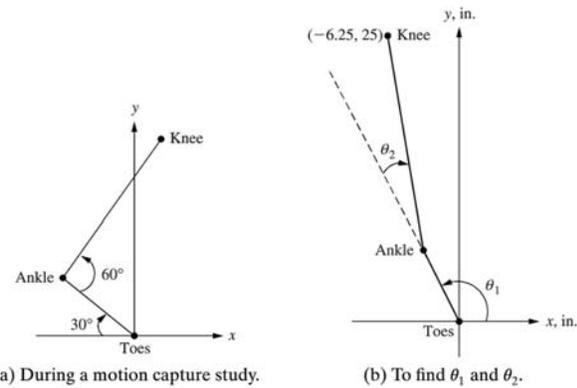
Figure P3.46 Position of the runner's leg.

(a) During a motion capture study.
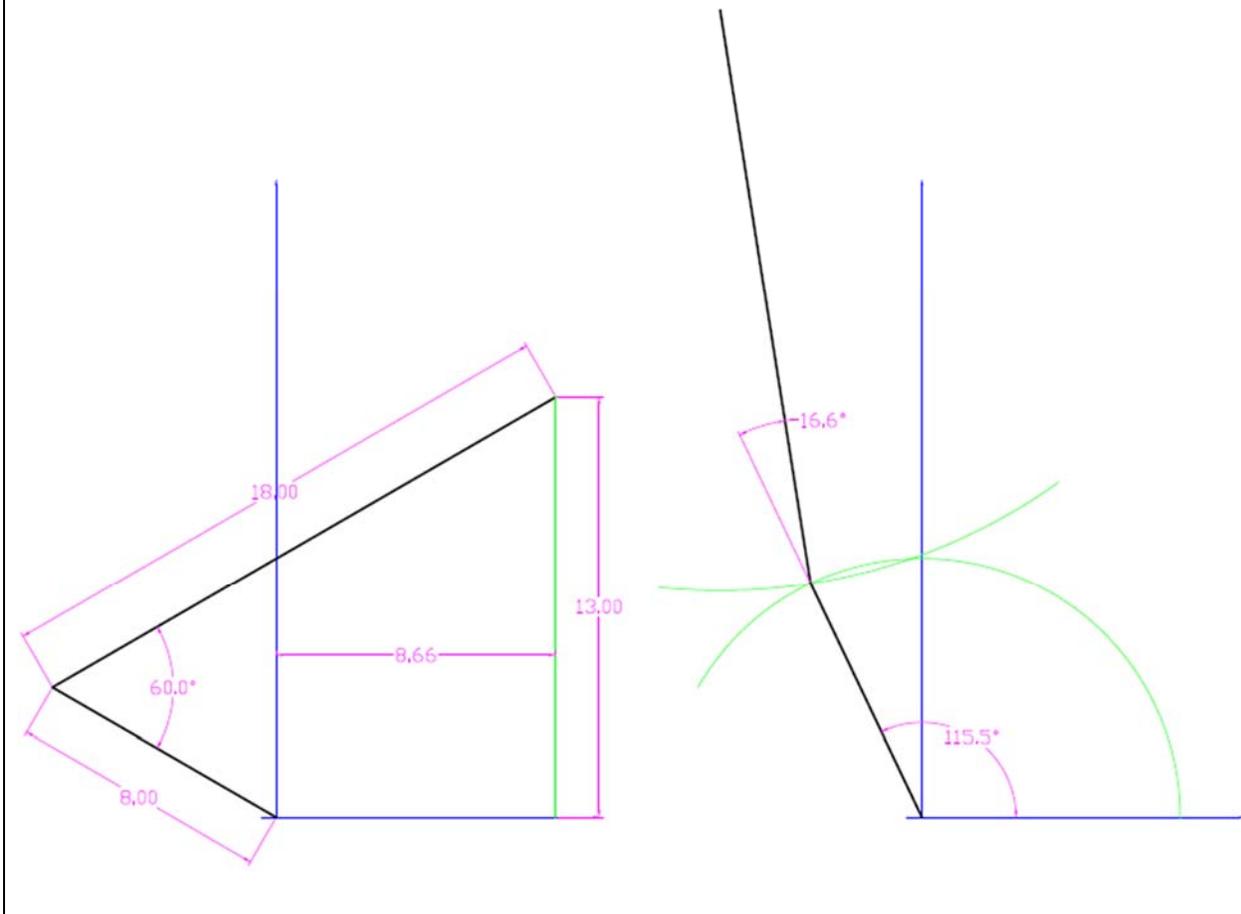
(b) To find θ_1 and θ_2.

Figure 6. Example problem 4 statement[1] and solution.

Example problem 4 is a common application considered in the text—the two-link planar robot. The solution on the left is referred in the text as the direct kinematics problem. Both link lengths and angles are given. In this case, the solution procedure is straight-forward.

The robot is drawn using relative coordinate entry, i.e. @8<150 and then @18<30. The position of the tip is indicted with dimensioning-linear.

On the right-hand side is the inverse kinematics problem. To solve this problem, two construction circles (shown in green) are drawn. (Note that the circles have been trimmed for clarity in the presentation.) The first circle is drawn with the center at (0,0) and radius of 8 (the link length) and then the second circle is drawn with the center at (-6.25,25), relative to the first point, with a radius of 18 (the link length).

Example Problem 5

Example problem is vector addition problem from statics.[5] The solution is determined using the parallelogram rule for vector addition. First, the two given vectors are drawn using relative polar coordinate entry. Then the two construction lines are drawn using relative polar coordinate entry. The starting point is found using Osnap-endpoint. The resultant vector is drawn using the line command and Osnap-endpoint. The arrow on the resultant vector is drawn with Polyline (start width = 0 and end width = 4).
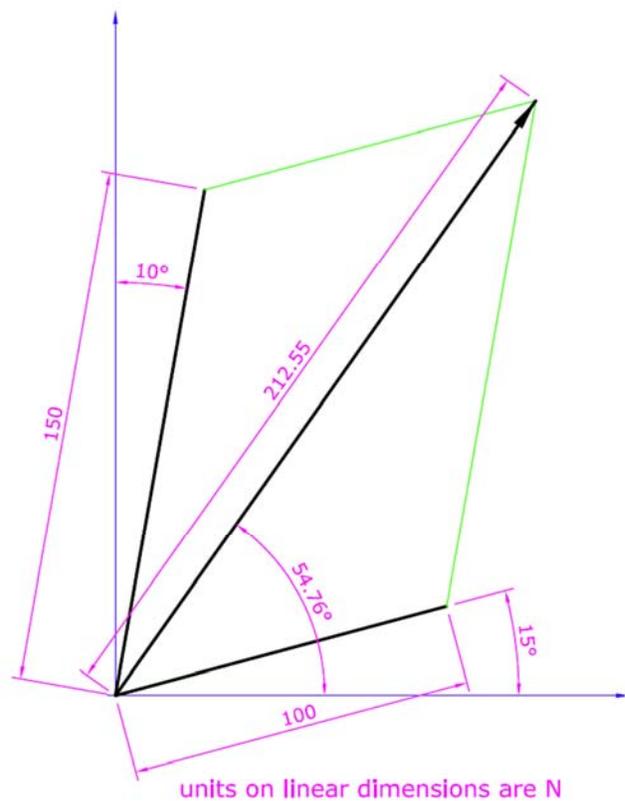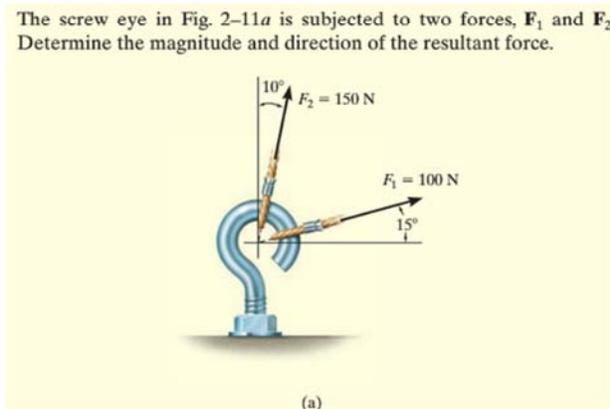

Figure 7. Example problem 5 statement[5] and solution.

Concluding Remarks

This paper describes how CAD software is used to solve trigonometry and vector problems in a first-year engineering course. The use of CAD software to solve these problems is part of an effort in this course to introduce students to multiple solution procedures which also includes paper-and-pencil trigonometric equations and spreadsheet representation. The belief is that multiple solution procedures will better connect with different types of student learners and promote a deeper level of understanding among all students. In addition, CAD software is used as one of several ways (others include paper-and-pencil and spreadsheets) to represent and communicate solutions to trigonometry and vector problems.

To measure the success of these efforts, two student learning outcomes, as shown in the table below, were assessed. Very preliminary results indicate that students feel that these outcomes are being achieved.

Table 3: Student learning outcome (SLO) measurement

Students who successfully complete this course will be able to…	score
Lecture SLO 7. solve and document the solution of problems involving new elements by using multiple approaches including visual problem solving and problem solving formalisms	3.3
Computer Lab SLO 4. create and use drawings and diagrams to solve a problem and to document its solution	3.2
Strongly agree = 4, agree = 3, disagree = 2, and strongly disagree = 1	

These results are the average of the responses for all students (over 100 students in five different sections) completing the assessment measure. More detailed assessment of these outcomes and other outcomes associated with the course is ongoing.

References

1. K. S. Rattan and N. W. Klingbeil, *Introductory Mathematics for Engineering Applications*, Wiley, 2015.

2. Howard Gardner, *Frames of Mind: The Theory of Multiple Intelligences*, Basic Books, 2011. First published in 1983. See also https://howardgardner01.files.wordpress.com/2012/06/faq_march2013.pdf

3. Stephen DeMeo, *Multiple Solution Methods for Teaching Science in the Classroom Improving Quantitative Problem Solving Using Dimensional Analysis and Proportional Reasoning,* Universal-Publishers, 2008.

4. D. Jonassen, J. Strobel, and C. B. Lee, Everyday Problem Solving in Engineering: Lessons for Engineering Educators, Journal of Engineering Education, 139-151, April 2006.

5. R.C. Hibbler, *Engineering Mechanics: Statics*, Pearson-Prentice Hall, 2011.

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