An engineering mathematics course project on modelling automobile suspension system

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Abstract

The importance of engineering mathematics can never be overemphasized for mechanical engineering students. Engineering mathematics provides students with mathematical tools for many other mechanical engineering courses and equips students with modelling skills to formulate engineering problems into mathematical forms. Another objective of this course at the Purdue University North Central is to teach students about how a major project is to be accomplished from start to finish.

A course project on analyzing the dynamic responses of an automobile suspension system has been designed based upon quarter-car model. This project helps students develop ability to apply engineering mathematical knowledge (differential equation, Laplace transform, and transfer function, for example) to identify, formulate and solve problems in the area of mechanical engineering. This paper describes the project assignment in detail, summarizes the outcomes of this course project, and discusses the relationship between this project and the mechanical engineering curriculum's educational objectives.

Introduction

With the background in calculus and differential equations, the junior students in the Mechanical Engineering Program at the Purdue University North Central take the engineering mathematics course which focuses on the theories and applications of ordinary and partial differential equations [Zill, 2010]. This course provides students with the necessary engineering mathematical knowledge for some core courses in Mechanical Engineering. For example, there are many applications of vector calculus in the Fluid Mechanics, applications of eigenvalue problems in the Vibration Analysis, and applications of various boundary value problems in the Engineering Numerical Methods (finite element method, for example). Also knowledge of Fourier series, Fourier transform, and Laplace transform improves students' abilities to analyze problems in the area of mechanical engineering.

Although realizing the importance of this engineering mathematics course, most students feel unaware of the practical applications of what they have learned in class.

This class project has been designed to build a bridge between the abstract mathematical theorems and practical mechanical engineering problems. The project focuses on the characteristic analysis of vehicle suspension system based on quarter-car model [Gillespie,1992]. One objective of the project is to help students develop ability to apply engineering mathematical skills (differential equation, Laplace transform, and transfer function, for example) to model and solve a spring-mass-damper system that is widely used in mechanical engineering. Another objective is to teach students about how a major project is accomplished from start to finish, prior to their senior design project. In this paper the project assignment will be presented in detail, along with a discussion of its outcome, and its contribution to the mechanical engineering curriculum's educational objectives.

Project Description

Quarter-car model is widely used in automotive engineering to simulate the dynamic behavior of vehicle suspension systems. The main function of suspension is usually simulated by spring and damper components which provide the necessary ride isolation at each wheel. Figure 1 shows a simple quarter-car model. The sprung mass (M) represents the mass of the vehicle supported on the suspension and the unsprung mass (m) is defined as the total mass of the parts being connected to the wheel directly. The stiffness and damping coefficient of the suspension are denoted, respectively, by K_s and C_s. K_t denotes the stiffness of the tire whose damping effect is ignored.

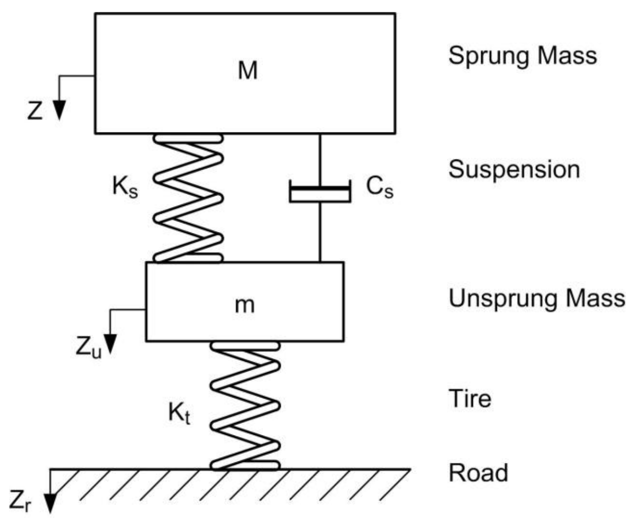

Figure 1 Schematic diagram of quarter car model

The dynamic behavior can be characterized most meaningfully by considering the input-output relationships. The input may be any of excitations (road roughness, deriveline excitation, excitation from the internal combustion engine, or excitation from the tire/wheel assembly, for example). The road excitation, Z_r, is chosen to be the input to the suspension system, and it will be transmitted to the occupants through the wheel/tire assembly, suspension system, frame, and cab. From the ride quality point of view, designers are mostly interested in the vibration of the sprung mass, which is used as the system output. The term "transmissibility" is usually adopted to represent the non-dimensional ratio of the response amplitude of the sprung mass to the excitation amplitude of the rough road.

The sprung and unsprung masses are known, while other system parameters including the stiffness of the spring components and damping coefficient of the dashpot are to be determined by students. The project is conducted based mainly upon the strategy shown in Figure 2. Students determine, through literature research, the appropriate ranges of the stiffness and damping coefficient according to the known sprung mass. Also they learn the assumptions upon which the quarter-car model is built, as well as the state-of-the-art of the research in this area. The literature review findings are summarized in their final reports. The differential equations for modelling the motions of the sprung mass and unsprung mass are derived from free body diagrams. This has been proven to be a critical component of the project. Students enhance their understanding of the differential equations from engineering point of view, instead of mathematics point of view. Compared to that in time domain, it is difficult for most students to understand and interpret the analysis results in frequency domain. Transmissibility is the non-dimensional ratio of response amplitude to excitation amplitude in a form of either displacement, or velocity, or accelerations. Laplace transform is applied to manipulate the derived differential motion equations to yield system transmissibility as a function of exciting frequency (Although students are familiar with the properties, theorems, and techniques associated with Laplace transform through class discussions and homework assignments, they are lack of the experience of applying this powerful mathematical tool to practical engineering problems). By plotting the transmissibility with respect to variable stiffness and damping coefficient in the predetermined ranges, students learn to relate the system response characteristics (e.g. natural frequency, under/critical/over damping) with the "coefficients" in the differential equations. Students may be excited to find that they can predict the system output (resonance, for example) based on the designed system parameters (e.g. mass, stiffness, and damping coefficient).

Project Management

The Purdue University North Central has taken the Association of American Colleges and Universities (AAC&U) Essential Learning Outcomes for its general education curriculum. One of the purposes of the Essential Learning Outcomes is to prepare students for gaining knowledge of intellectual and practical skills, including inquiry and analysis, critical and creative thinking, written and oral communication, team work and problem solving. This project assignment helps students achieve the course outcomes in developing students' ability to apply knowledge of mathematics, science, and engineering in the field of mechanical engineering and to identify, formulate, and solve mechanical engineering problems.

The class is divided into four teams of at least three students each. Team project is superior to individual project in that more and more engineering research problems require collaboration of investigators with different expertise. It also provides students an opportunity to practice in the professional conferences, meetings, and consulting activities. Another advantage of team work over individual work is the improvement of students' skills for dealing with personality clashes and making decisions efficiently. Each team is formed voluntarily without any interference from the instructor and it is completely the junior students' responsibility to allocate workloads among the team members. It is found that the depth and completeness of the project research partially depend on the research strategy, organization, and time spent on the project.

Grading for each individual is based upon the project report and oral presentation. Recommended outline for the final report includes a brief introduction and statement of objectives, a complete sufficiency analysis, and a discussion of the results. The programming source codes and supporting analyses may be arranged in the appendix section. Students are encouraged to prepare their reports in a form that conforms to the same standards as the project proposal, including the specifications on the font, margins, page numbers, paragraphs, and section. Equations should be centered with sequential number in parenthesis. Figures and Tables must be captioned and sequentially numbered. The oral presentation plays an important role in the project assignment. Each team has ten minutes for presentation in which at least two minutes should be reserved for audience questions. Team members cooperate in the presentation, illustrating their work on the research background, analysis details, and concluding remarks. Other teams may put forward questions concerning the presenters' analysis and results, or they may challenge the presenters with questions from their own studies. Each student is graded individually by other students, including his/her teammates, mainly based upon the logical sequence organized in the presentation, professional behavior during presentation, and presenter's responses to audience questions.

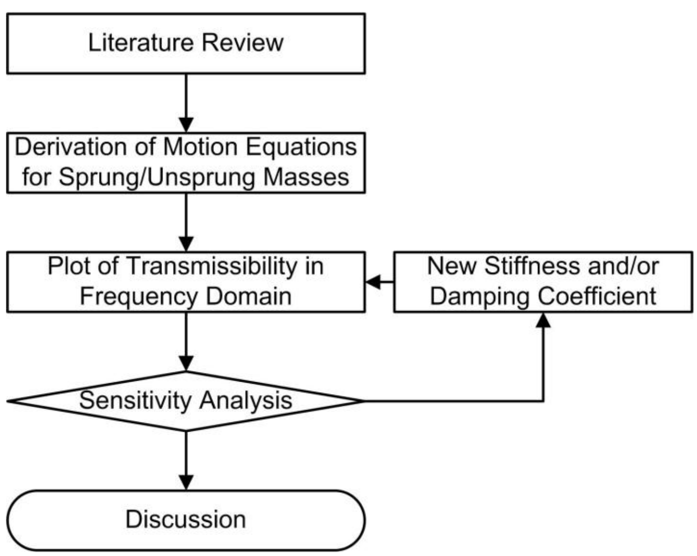

Figure 2 Flowchart for conducting project research

Conclusions

This paper documents the author's attempt to introduce a course project in engineering mathematics teaching. The project is about the dynamic behavior analysis of a vehicle suspension subsystem. It is believed that students have enhanced their understandings of the differential equations through modelling the suspension behavior, Laplace transform through transferring the analysis results from time domain to frequency domain, and the transfer function through analyzing the sensitivity of the system characteristics with respect to the system parameters. Also this project is served to teach students, prior to their senior design project, how to accomplish a project from start to finish. Detailed descriptions about the course project are presented, along with a discussion of its contribution to the mechanical engineering curriculum's educational objectives.

Bibliography

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2. Zill, D. and Wright W. 2010, Advanced Engineering Mathematics, 4th edition, Jones and Bartlett Publishers