# What is the Role of Mathematical Modeling in Core Mathematics Courses for Engineering Students?

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**Abstract**

The importance of mathematics to engineering is not in dispute. Engineering majors are required to study far more mathematics than most other majors are required to study. There often are, however, disagreements between mathematicians and engineers on the best way to teach mathematics to engineering students. We describe possible reasons for these disagreements. We provide an example of how mathematical modeling can be used as a bridge to an abstract mathematical concept known to mathematicians as the implicit function theorem, but referred to by engineering students as "you got to have as many equations as unknowns." We illustrate how mathematical modeling highlights mathematical subtleties often overlooked by students.

**Key Words**

Interdisciplinary Approaches, Education Methods

# What is the role of mathematical modeling in core mathematics courses for engineering students?

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## Introduction

Ellis et al. [2] conducted a survey in which 96 practicing engineers were asked if they used or required conceptual understanding of specific topics commonly taught in core mathematics courses for engineering students. Ellis et al. conclude "Math and basic science are certainly the foundations of any engineering program. This fact will not change in the foreseeable future." Mustoe states "The central role of mathematics in engineering has long been accepted. That it is an essential tool for describing and analyzing engineering processes and systems is not in dispute." [7] That mathematics is a fundamental part of engineering should not be surprising. The laws of nature are written in the language of mathematics. Mathematics also enables precise representation and communication of knowledge.

Engineering students often have difficulty learning and using abstract mathematical concepts.[10] Problem solving to many of them means searching for the proper formula and plugging in the necessary numbers.[12] Willcox and Bounova summarize a study at MIT which they used to identify "barriers to deep mathematical understanding." The level of the mathematics skills of sophomores and juniors was identified as a problem by a number of faculty in the Department of Aeronautics and Astronautics. [14]

Based on my own experience teaching mathematics, I believe engineering students are not learning to use mathematics to its full potential. Consider the following diagnostics test questions given to sophomore engineering majors at Rose-Hulman Institute of Technology in the Fall of 2003. (Rose-Hulman Institute of Technology is an undergraduate institution in the United States specializing in engineering education.)

*Question 1*:

Jack is four years older than twice Jill's age. Jill is one year younger than half Jack's age.
How old are Jack and Jill?

*Question 2*:

Jack is four years older than twice Jill's age. Jill is two years younger than half Jack's age.
How old are Jack and Jill?

Of the 53 students who took the test, only 4 answered question 1 correctly and only 3 answered question 2 correctly. I believe the reason most students missed these questions is because of the way mathematical modeling is taught in mathematics courses. Most were able to obtain the correct system of equations. Below is a sample of the incorrect answers given.

*Question 1*

Jill = 6, Jack = 16 (nope)
0 = 2 um..no.., 0 = 1 um..no..
2 = 4 ?
I have no idea, I am confused.

*Question 2*

Jack = 10, Jill = 3
They are the same age.
They can be any age.
No answer. Can't be solved.
x = x, Therefore, works in all cases.
Ugh..Not a good start!   eh!

The few nearly correct and correct answers given are listed below.

*Question 1*

no solution.
Not possible, tried trial and error, both equations don't match.
This situation can not happen.
The two equations don't match.

*Question 2*

can be numerous answers. Tried trial and error method.
can not determine with info.
any age such that one of the conditions is true?

In this article, I will explain why I feel mathematical modeling is not receiving the attention it deserves in core mathematics courses for engineering students. I will also provide an example of how mathematical modeling can be used as an organizational framework for a core sophomore mathematics course.

## Learning Objectives

Mathematics and engineering faculty often disagree on the appropriate learning objectives for core mathematics courses. Like courses in the humanities, core mathematics courses have broader objectives than just supporting engineering programs. Most mathematics educators want students to develop an understanding and appreciation of mathematics, its precision, its rigor, its generality and the process by which it is developed. Engineering educators, on the other hand, are primary concerned with mathematical manipulation skills and solution techniques. On the teaching of mathematics to engineering students, Mustoe writes:

> *The issue is perceived by many engineers and mathematicians to be a long-running source of tension. Part of the problem has been that…(those)…teaching the mathematics component of the engineering degree tended to provide a watered-down*

*version of modules…being taught to their own students. Mathematics, it was argued, was a subject in its own right and any applications should be left to the engineering lecturers.*[6]

Strandler , who teaches engineering, writes:

*…a provocative comment frequently heard among professors of engineering: "Proofs of theorems and discussions of axioms, postulates, etc. should receive minor treatment (but not eliminated! )…few successful engineers are able to state Rolle's Theorem in calculus. Is such material really critical?*

*While most engineers do few, if any, proofs, they do many derivations and mathematical operations. Students should receive extensive practice in doing derivations.*[12]

The failure of some mathematics faculty to stress the importance of using correct units is a source of frustration for some engineering faculty. Rowland observed in a study of beginning engineering students that few students who completed a first year introduction to differential equations understood that the units in each term of a differential equation must be the same.[9]

Confusion caused by terminology is another source of friction between mathematics and engineering educators. Worse yet, at many schools, little formal communication exits between mathematics and engineering faculty. Typically, students are expected to translate terminology and make the connections between mathematics and engineering courses on their own.

Current mathematics textbooks for engineering majors are not particularly helpful. Most are encyclopedic in their coverage of mathematics. Most also have a level of mathematical rigor which far exceeds the abilities of the majority of engineering students.

Engineering educators sometimes fail to appreciate the hierarchal nature of mathematics. Foundations must be mastered first. In a survey of 96 practicing engineers,[2] numerical methods was identify as the most used mathematical concept/topic and infinite series the least used concept/topic. Most survey respondents felt that, with the exception of infinite series, conceptual understanding of core mathematics concepts was important for their jobs even if the associated calculation techniques were not used very often. Interestingly, the respondents may not have been aware that infinite series are a key part of the theoretical justification for many numerical methods.

Strandler believes proofs of theorems should receive minor treatment. However, he believes engineering students should receive extensive practice doing derivations. Problems with more unknown parameters and fewer numbers should be given to students. In an editorial on engineering mathematics, Standler states "It is critical that engineering students learn to visualize abstract concepts…Mathematics courses should be abstract and general." [12] Boyce, one of the authors of a well known differential equations textbook, observes, "Details of mathematical procedures and algorithms are rapidly forgotten unless they are used frequently, but underlying concepts and ideas become part of an individual's mindset and are always available…." [1]

We must guard against mathematics being perceived by students as simply a collection of abstract techniques.[13] We need to find the right balance between practical applications of mathematics and in-depth understanding.[10] These observations suggest that core mathematics courses for engineering students be organized around fundamental concepts which students can appreciate as having lasting value.

The learning objectives of core mathematics courses at Rose-Hulman can be put into three categories: (1) Content Objectives: Students should learn fundamental mathematical concepts and how to apply them. (2) Skill Objectives: Students should learn critical thinking, modeling/problem solving and effective uses of technology. (3) Communication Objectives: Students should learn how to read mathematics and use it to communicate knowledge. Even though modeling is explicitly stated as a learning objective, the way modeling is used in the mathematics curriculum differs significantly from the way it is used by engineers in practice.

**Mathematical Modeling**

Mathematical modeling is a key part of engineering. Gainsburg conducted a detailed ethnographic study of the everyday problem-solving activities of two practicing structural engineers. Gainsburg concluded:

> *Modeling was found to be central and ubiquitous in the engineers' work…transforming hypothetical structures into mathematical or symbolic language for the purpose of applying engineering theory is the heart of their profession. Modeling…drives the bulk of their sense-making activity and defines professional expertise.*[3]

In 1995, Rose-Hulman Institute of Technology began offering a new core engineering course sequence in which students learn to formulate mathematical models from first principles using conservation laws. Faculty involved in developing the course sequence identified modeling as a key engineering activity.[8]

Mathematical modeling is an iterative process. Models are refined and validated before they are used. Typical steps in the modeling process as outline in Gainsburg's paper [3] are:

1. Identify real-world phenomenon.
2. Idealize phenomenon.
3. Express idealized phenomenon mathematically.
4. Perform mathematical manipulations (i.e. "solve" the model).
5. Interpret mathematical results in real-world terms.
6. Test interpretation against reality.

Mathematical modeling in core mathematics courses differs significantly from the mathematical modeling practiced by engineers. In core mathematics courses, a trivialized version of modeling centering on step 4 above is used. Steps 1–3 are short-circuited by explicitly stating assumptions that students are expected to use in formulating a model. Step 6 is skipped altogether. Interestingly, Gainsburg identifies keeping track of changing assumptions as models are refined—as one of the most challenging aspects of the modeling practiced by the structural engineers in his study.[3]

In most mathematics courses, modeling is used as a means to teach mathematical concepts rather than to make students proficient modelers.[15] Fundamental mathematical concepts are difficult for students to understand, appreciate and apply. Sazhin provides evidence that engineering students find it much easier to learn physical concepts than mathematical ones.[10] Sazhin also points out that it should not be taken for granted that engineering students understand the need to study mathematics in the first place.

In the mathematics curriculum, mathematical modeling is primarly used as a way to communicate abstract ideas and generate interest and motivation. In the words of Laver, "…modeling teaches a colloquial mathematics…" [4] Some may view the term "colloquial mathematics" as denoting an inferior form of mathematics. However, we should remember that most engineers use mathematics in a "colloquial" manner.

## Colloquial Mathematics

In this section I provide an example of "colloquial mathematics" which I believe is of such importance that it should be called The Fundamental Theorem of Mathematical Modeling. I will contrast this colloquial mathematics with the mathematical theorem on which it is based and point out subtleties typically missed by students.

Mathematics models used in mathematics courses are typically "well-posed." That is, the associated equations have unique solutions. It is likely the lack of experience with "ill-posed" models that explains the difficulty students had with the Jack and Jill questions described in the introduction.

The following theorem is of fundamental importance in mathematical modeling.

> Fundamental Theorem of Mathematical Modeling
>
> *The number of free variables in a mathematical model equals the number of variables in the model minus the number of independent equations.*

Stated in a more rigorous fashion, this theorem is known to mathematicians as the Implicit Function Theorem and is recognized to be one of the pillars of mathematical analysis. In an undergraduate linear algebra class, students are taught a version of this theorem which reads:

> *The dimension of the nullspace is equal to the dimension of the domain minus the dimension of the range.*

Not surprisingly, many if not most students do not make the appropriate connections to mathematical modeling. Instead, they learn, probably through experience, an incomplete version that reads *you have to have the same number of equations as unknowns*. Missing is the important requirement that equations must be independent.

## Modeling as a Framework

What would a core mathematics course look like if mathematical modeling were used as an organizing framework? Consider the following five week course segment on the Fundamental Theorem of Mathematical Modeling. The course centers on step 4 of the modeling process outlined earlier. (I should stress that the primary goal of this course segment is to teach mathematical concepts, not modeling.) The following mathematical concepts are covered.

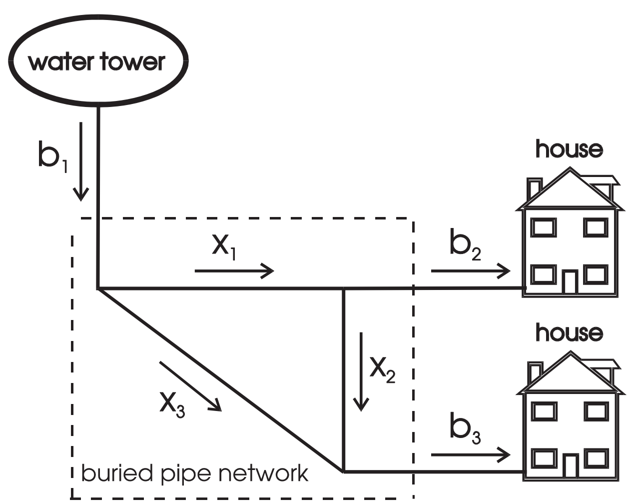
1. independent equations
2. free variables
3. inconsistent equations
4. necessary conditions for solutions to exist
5. least squares solutions

Medley lists five characteristics which predict when the application of mathematical modeling is likely to be successful.[5]

1. The application involves a small number of significant factors.
2. A sufficient number of already well-corroborated laws relating those factors exists.
3. The modeler understands the subject matter describing the application.
4. The mathematics needed falls exclusively within the area of competence of the modeler.
5. Sufficient time is available.

These five characteristics are also useful in identifying good problems for use in core mathematics courses. Consider the following problem.

> Problem:
>
> *Consider the pipe network shown below. Assume the flow rates $b_1$, $b_2$, and $b_3$ are known. Determine the flow rates $x_1$, $x_2$, and $x_3$ in each of the buried pipes.*




The problem described above satisfies Medley's criteria 1, 2 and 3. My experience using this problem in a sophomore level mathematics course has shown that students have no difficulty use conservation of mass (water) to derive the three equations which relate the three known quantities $b_1$, $b_2$, and $b_3$ to the three unknown quantities $x_1$, $x_2$, and $x_3$. The equations are:

$$x_1 + x_3 = b_1$$
$$x_1 - x_2 = b_2$$
$$x_2 + x_3 = b_3$$

Since the purpose of the problem is to teach new mathematical concepts, it should be expected that criteria 4 is not met. Criteria 5 will be discussed in the next section.

Students are surprised to find that the above equations have many solutions. This is because students are rarely assigned ill-posed problems in their core mathematics courses. The above problem motivates a careful study of what it means for equations to be independent of one another. The necessary linear algebra is introduced in a "colloquial" fashion, treating linear functions, equations and spaces as mathematical tools for understanding the Fundamental Theorem of Mathematical Modeling, all in the context of the above problem.

The above system of equations contains only two independent equations. The Fundamental Theorem of Mathematical modeling implies the model has one free variable. A physical interpretation of the free variable resulting from the "missing equation" is sought. The free variable is shown to describe flow in the loop in the network. This leads to the "discovery" of pressure drop across the pipes as the information missing from the model. Students are again surprised to find that the problem doesn't end here. A reference level for pressure must also be established to uniquely determine the pressure at each node in the network. Interestingly, the flow rates in the buried pipes are shown mathematically to be independent of this reference level.

Once students are confident solving pipe networks, a network with $b_1 \neq b_2 + b_3$ is given to them. Again, they are surprised when they are unable to compute a solution. Networks without solutions motivate a study of what it means for equations to be inconsistent. Necessary conditions for a solution to exist are mathematically derived and shown to be nothing more than conservation of mass (water) for the network as a whole.

Finally, least squares solutions are motivated as a method for obtaining reasonable solutions when measurement or rounding errors result in violations of conservation of mass. The least squares method is interpreted as an intelligent way to adjust the values of the flow rates $b_1$, $b_2$, and $b_3$ so that the network as a whole satisfies conservation of mass.

**Less is More**

Modeling is an activity best learned as an activity rather than taught as a concept. Medley writes:

> *…mathematics…and modeling are necessarily competitors for time, notwithstanding…that both the roots of mathematics and the fulfilment of mathematics are to be found in modeling*.[5]

This observation of Medley points to the main obstacle in using mathematical modeling as an organizational framework for mathematics courses. Is there enough time in the current curriculum?

It is much easier to add content than remove content from the curriculum, especially in the multi-section courses typically taken by engineering students. Small details the curriculum development forces which act to create too much content and a subsequent shallow treatment of topics.[11] Townend writes "today's densely packed syllabuses can lead to a lack of clear understanding by the students and an inability to apply the techniques appropriately, and suggests that a firm understanding of a reduced core would be preferable.[13] The increased use of computer algebra systems such as Mathematica and Maple has led to a decreased emphasis on methods of integration in calculus courses and solution techniques in differential equations courses. Perhaps these trends present an opportunity to restructure the core mathematics curriculum.

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