A Unique Pendulum Impact Apparatus Capstone Design Project for a Hands-On Senior-Level Laboratory Design Experience
Yin-ping (Daniel) Chang, Oakland University

Abstract
This paper describes a particle kinetics work/energy and impulse/momentum measurement experiment for the senior level Engineering Mechanics – Dynamics course. This course introduces students to the fundamental principles of kinematics and kinetics of particles and rigid bodies, including displacement/velocity/acceleration kinematic relationships and kinetic analyses through Newtonian laws of motion, work/energy conservation laws, and impulse/momentum approaches. It has been the Mechanical Engineering Department’s philosophy that theory learned in the classroom be augmented by experiential knowledge gained by laboratory experience. In this light, hands-on laboratory experiments have been developed that are integrated with the course material. This paper presents a unique experimental apparatus, designed and built at Oakland University, by senior-level students involved in a design project. The purpose is to introduce students to particle kinetics properties measurement techniques to measure particles’ velocities, energy transfer and dissipation, and the coefficient of restitution during impact phenomena in a pendulum impact system. The experiment covers basic concepts of kinetics of particles, specially focusing on impulse/momentum related principles. Two objects were used in this impact apparatus. One object was set up as a pendulum, being raised up and swung down to impact a stationary object. The first object was raised up to store the gravitational potential energy, and then swung down to transform the gravity potential energy into kinetic energy; this object then impacted the other object, transferred part of the momentum to the other object, the other object gained the momentum and transferred it into kinetic energy. The energy was then dissipated by friction when the object traveled on a flat surface. The students were asked to validate the particle kinetics law of conservation of energy and impulse/momentum principles. Results of the students’ experiences will be presented in this paper.

Key Words
Engineering Curricula
A Unique Pendulum Impact Apparatus Capstone Design Project for a Hands-On Senior-Level Laboratory Design Experience

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1. ABSTRACT

This paper describes a particle kinetics work/energy and impulse/momentum measurement experiment for the senior level Engineering Mechanics – Dynamics course. This course introduces students to the fundamental principles of kinematics and kinetics of particles and rigid bodies, including displacement/velocity/acceleration kinematic relationships and kinetic analyses through Newtonian laws of motion, work/energy conservation laws, and impulse/momentum approaches. It has been the Mechanical Engineering Department’s philosophy that theory learned in the classroom be augmented by experiential knowledge gained by laboratory experience. In this light, hands-on laboratory experiments have been developed that are integrated with the course material. This paper presents a unique experimental apparatus, designed and built at Oakland University, by senior-level students involved in a design project. The purpose is to introduce students to particle kinetics properties measurement techniques to measure particles’ velocities, energy transfer and dissipation, and the coefficient of restitution during impact phenomena in a pendulum impact system. The experiment covers basic concepts of kinetics of particles, specially focusing on impulse/momentum related principles. Two objects were used in this impact apparatus. One object was set up as a pendulum, being raised up and swung down to impact a stationary object. The first object was raised up to store the gravitational potential energy, and then swung down to transform the gravity potential energy into kinetic energy; this object then impacted the other object, transferred part of the momentum to the other object, the other object gained the momentum and transferred it into kinetic energy. The energy was then dissipated by friction when the object traveled on a flat surface. The students were asked to validate the particle kinetics law of conservation of energy and impulse/momentum principles. Results of the students’ experiences will be presented in this paper.

2. OBJECTIVES

With all engineering classes, labs play an important role in relating the theory discussed in class to practical applications. However, ME321 Dynamics and Vibration at Oakland University, lacks this core component which helps bridge the theory and the practical aspects. As a result, a lab experiment was designed, built, and tested that can be integrated into the curriculum directly. The objective of this lab was to introduce particle kinetics, especially focuses on work/energy and impulse/momentum concepts and approaches. From the apparatus designed, two blocks, aluminum and steel were impacted by a pendulum attached with a steel cylinder. The experiment was conducted with the pendulum set to 30° and 50° angles. The distance traveled by
the block upon impact was measured and recorded. Initial and final velocities were calculated for the block and cylinder, which were used to determine coefficient of restitution. This experiment demonstrated two different approaches need be used at the same time to evaluate problems involving collisions.

3. DESIGN OF EXPERIMENT

The basic concept behind this laboratory was to develop a user friendly lab that would help enrich the material that is learned in class. The idea was to create a pendulum that would swing freely and contact another object, in which the traveled distance could be measured. There were a few specified design constraints that had to be considered. The overall structure had to be light in weight and smaller in size to allow for easy transportation, and it could last over time and continue to be effective tools for enhancing the lectured material.

Computer aided design software was used in the process of designing the lab. AutoCAD 14 and AutoDesk Viz 4.0 were tools used in creating the laboratory models. AutoCAD 14 was used to accurately dimension each component and to give final schematics of the completed structures. AutoDesk Viz 4.0 is a type of software that basically brings AutoCAD to life. It was used to provide a complete 3-D model including accurate surface textures and colors. After the model was created, this software ran a simulation of the experiment. This is an effective aid to give a general idea of how the laboratory is to be run.

Figure 1 - The Lab ISO View
Friction tests were conducted between two identical aluminum surfaces and also between a steel and aluminum surface. The registered ASTM D 1894 standard friction test yielded results that were used for laboratory calculations. The sample blocks were conditioned at standard laboratory conditions of 21±2°C and 50 ± 5% relative humidity for a minimum 24 hours prior to testing. The coefficient of friction test was performed on an Instron machine using TestWorks 3.1 software. A desired material block was placed on a desired surface. The block was then pulled at a constant speed of 150 mm/min to determine the coefficient of friction between the two materials. This was repeated two more times (total of three times) and the average value was taken in order to determine the theoretical coefficient of friction. The coefficient of friction is a resistive force encountered between two objects either stationary or in motion. Since this lab analyzed the motion of objects, it was necessary to determine the dynamic coefficient of friction.
Figure 3 – Performing the ASTM D 1894 Standard Friction Test
Table 1 – A Sample of Coefficient of Friction Test Data

Report Date: Dec 9, 2003

Reliable Analysis, Inc.
379 Inusco Ctr.
Troy, MI 48083
Tel: (810)588-9770

Sample ID: Steel Block on Alum Surface
Method: COF - ASTM D 1894
Data path: C:\TW300

Test Date: Oct 25, 2003
Operator ID: HUY

<table>
<thead>
<tr>
<th>Peak Load N</th>
<th>Max Ld S-P N</th>
<th>Avg Ld S-P N</th>
<th>Static COF (none)</th>
<th>Dynamic COF (tone)</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>2.7</td>
<td>2.668</td>
<td>2.192</td>
<td>0.355</td>
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<tr>
<td>2</td>
<td>2.5</td>
<td>2.463</td>
<td>2.019</td>
<td>0.297</td>
</tr>
<tr>
<td>3</td>
<td>2.3</td>
<td>2.309</td>
<td>1.664</td>
<td>0.256</td>
</tr>
</tbody>
</table>

Mean 2.5     2.480    1.958     0.303     0.315
Sdv 0.2      0.180    0.269     0.050     0.043

Specimen Comments:

<table>
<thead>
<tr>
<th>Calculation Inputs:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elong Point 2</td>
</tr>
<tr>
<td>Elong Point 3</td>
</tr>
<tr>
<td>Sled Weight</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test Inputs:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crosshead Speed</td>
</tr>
<tr>
<td>Ext Limit HI</td>
</tr>
</tbody>
</table>

For all components, clearance holes were drilled and reamed (smoothing radius of the hole) for screw and dowel placement. The first step was to machine two identical blocks, one of aluminum and one of steel. These will be the blocks that are struck by the pendulum. Also, a steel cylinder was fabricated to a specified dimension. This is the component that will be striking the identical blocks of steel and aluminum. A tooling plate (alum. w/ parallel surfaces) was used as the base. Jig feet (hardened rubber pads) were screwed to the bottom of the tooling plate to enable the lab experiment to be transported whenever needed.

Two aluminum risers were setup, then the steel rod is mounted between the risers. The steel cylinder was then connected to the lightweight threaded rod. A degree indicator was mounted on the side of the aluminum riser. This will allow the pendulum to be set to a desired angle prior to deployment.
Upon completion of the lab, testing was done to check for possible problems. A few glaring problems were identified. A student setting the pendulum to a certain degree and releasing the cylinder had no way of obtaining the desired angle every time constantly. In addition, the block sitting stationary on the apparatus was not set up in the same position twice. This offsetting of the stationary block yielded an oblique impact between the two objects. These variables caused enormous errors during testing. The release of the pendulum and the placement of the block needed to be regulated. Fabricating a v-notch block that the pendulum could rest on created a stable starting point. To release the pendulum the v-notch block was simply pulled out and the pendulum could swing straight and free. The placement of the block was constrained by grinding a thin piece of steel down to almost nothing. The piece of steel had a portion, the width of the block, machined out of it; therefore the block could rest against the piece of steel perfectly. This design iteration constrained the movement of the block. After these quality issues were addressed, lab was tested again, and now it met the R & R (Repeatability and Reliability) specifications.

5. ANALYSIS OF THE EXPERIMENT

The discrepancies between theoretical and experimental velocities after impact can be attributed to the assumption that the coefficient of restitution was equivalent to 1. From analysis of the experimental data, the coefficient of restitution was shown to be not exactly 1. This can be
attributed to the impact velocity, size, shape, and temperature causing variations. In addition, since the coefficient of restitution was not exactly 1, there was energy loss during impact between the cylinder and block. Another source of error can be attributed to oblique central impact of the cylinder and block. If the cylinder and block collide where the line of impact is not central then the collision is considered to be eccentric (oblique impact). Eccentric collisions reduce the distribution of energy which in turn reduces the velocities. Another contributing factor to the discrepancies in the calculated velocities was due to friction between the contact surfaces. This variation can be contributed to the different forces inflicted on the blocks caused by variations in speeds at which the system is initially run at. Secondly, surface defects played a role in the discrepancies with each trail when determining the coefficient of friction. The surface defects attributed to restricted motion on the block as it travels the length of the apparatus, implying that speed isn’t constant. But in order to determine the coefficient of friction, it is necessary to have a constant speed. As a result, the tabulated values needed to be calculated for each mass tested in order to reduce the percent error from these discrepancies.

6. CONCLUSION

It was concluded that this experiment will be beneficial to reinforcing the theory for dynamics. This lab provided a hands on approach as well as an opportunity to apply the theoretical equations and concepts. The experiment demonstrated the reliability and repeatability needed to accurately portray the situations discussed in class. After running the laboratory students will have exercised their general knowledge of physics and dynamics in real life situations and draw upon their own conclusions as modes for interpretation. Possible improvements could include the installation of a force gage allows more direct parameters of the object to be measured during impact; a digital angle measurement device could provide a more accurate reading.

7. ACKNOWLEDGEMENT

The author would like to acknowledge four students, Mr. Hanh Ho, Huy Dang, Michael Withee and Robert Manoni who have participated in this particular design project in fall 2003 semester at Oakland University. Their enthusiasm, creative thinking, and inquiring questions during their attempts to synthesize better designs, continually fuels the enthusiasm as teachers to discover and develop new ideas and methods to enhance our effectiveness as engineering educators.

Author’s Brief Biographical and Contact Information

Yin-ping (Daniel) Chang, Ph.D., he received his Ph.D. degree in 2002 and continues his research as an assistant professor at Oakland University, Rochester, Michigan. His current research interests include vehicle/tire dynamics, FEA computational solid mechanics, biomechanics, machine dynamics, machine design, and classical mechanism synthesis and analysis. E-mail address: ychang@oakland.edu, website: www.oakland.edu/~ychang.
ME 321: Dynamics and Vibrations

Laboratory Experiment #5 Assignment: Lab Handout

Particles Kinetics – Work/Energy and Impulse/Momentum Approach:

An Impact Pendulum

Objective:
The purpose of this experiment is to introduce the principles of work/energy and impulse/momentum.

**Theory:**

This lab experiment is based upon the *Work/Energy* (1) and *Impulse/Momentum* (2) *Principles*.

\[
T_i + V_i + U_{i \rightarrow f} = T_f + V_f \tag{1}
\]

\[
M_A V_i + \int F \, dt = M_A V_f \quad \text{(for individual particle)} \tag{2a}
\]

\[
M_A V_{Ai} + M_B V_{Bi} = M_A V_{Af} + M_B V_{Bf} \quad \text{(for multi objects)} \tag{2b}
\]

**Procedure:**

1. Measure the masses of both block A’s, and rod length L.  
   *(Note: The rod length is from the center of the shaft to the center of the cylinder B. Why?)*

2. With the experimental apparatus provided, set up the experiment as shown in the sketch.

3. Place the block (aluminum or steel) provided at the scribed starting position.

4. Place the V-notched block in recess area.

5. Rest the pendulum on V-notched block.

6. Remove the V-notched block to deploy the pendulum.  
   *(Note: Remove V-notched block quickly and smoothly, practice several times before you take the measurements. Make sure to remove the V-notched Block consistently.)*

7. Measure the distance traveled by the block and the angle of the pendulum before and after the impact.  
   *(Note: The distance traveled by the block should be measured from the center of the blocks no matter the blocks rotate or not.)*

8. Run this experiment 10 times and take the average measurements.

9. Repeat the experiment with another angle of the pendulum and then another block.

**Report & Discussion:** (For all 4 cases.)

1. Find the velocity of the cylinder B before and after impact.

2. Find the velocity of the block A right after the impact.
3. Determine coefficient of restitution.

4. Determine coefficient of friction between the block and the apparatus surface.

5. Discussions.
ME 321 : Dynamics and Vibrations

Laboratory Experiment #5 Assignment: Example Lab Report

Particles Kinetics – Work/Energy and Impulse/Momentum Approach:

An Impact Pendulum

Submitted to: Prof. Yin-ping Chang

Prepared by:

Hanh Ho
Huy Dang
Michael Withee
Robert Manoni

Introduction

The objective of this experiment was to introduce the concepts of work/energy and impulse/momentum methods. From the apparatus provided, two blocks, aluminum and steel, were impacted by a pendulum attached with a steel cylinder. The experiment was conducted with the pendulum set to 30° and 50° angles. The distance traveled by the block upon impact was measured and recorded. Initial and final velocities were calculated for the block and cylinder, which were used to determine coefficient of restitution.

Theory

This experiment entailed Work/Energy Principle and Impulse/Momentum Approach. Work is defined as:

\[ W = F \cdot d \vec{r} \]  

(1)
Basically, there are three forms of work/energy: potential energy \( (V) \), kinetic energy \( (T) \), and a general energy format \( (U_{1 \rightarrow 2}) \), where the work is done on an object from some initial position, 1, to some final position, 2. Potential Energy is a form of energy that is based on stationary movement. As an object is positioned vertically (farther from the earth’s surface) gravitational forces take into effect, as shown in Figure 1.

The gravity potential energy concept is shown in Figure 1 and can be described as:

\[
V_{1 \rightarrow 2} = \int_{1}^{2} F \cdot dr = \int_{1}^{2} (-mg) \cdot dy \\
V_{1 \rightarrow 2} = -mg(y_2 - y_1) \\
V_{1 \rightarrow 2} = -mgh 
\]  

(2)

The kinetic energy can be described as:

\[
\vec{F} = ma = m \frac{d\vec{v}}{dt} = m \frac{d\vec{v}}{dx} \cdot \frac{dx}{dt} = m\vec{v} \frac{d\vec{v}}{dx} \\
\int_{x_1}^{x_2} F \cdot dx = \int_{v_1}^{v_2} mv \cdot d\vec{v} \\
T_{1 \rightarrow 2} = \frac{1}{2} m(v_2^2 - v_1^2) 
\]  

(3)

Lastly, a relationship can be derived between the different forms of work done on the object. This relationship can be described as the Law of Conservation of Energy. This states that the total mechanical energy of the object at position 1, is equal to the total mechanical energy at position 2, and is illustrated below:

\[
T_1 + V_1 + U_{1 \rightarrow 2} = T_2 + V_2 
\]  

(4)

The principles of impulse/momentum are obtained by the integration of Newton’s second law with respect to time instead. The impulse/momentum approach is useful in solving problems in
which the velocities of a body at two different instants are to be related and the forces involved can be expressed as functions of time.

\[
\vec{F} = m\dot{\vec{a}} = \frac{d}{dt} m\vec{v} \\
\int Fdt = \int dm\vec{v} \\
\vec{m}\vec{v}_1 + \int_1^2 Fdt = m\vec{v}_2 \\
\vec{m}\vec{v}_1 = m\vec{v}_2 \quad \text{when} \quad \int_1^2 Fdt = 0
\]

(5)

Where the force, \( F \) is an internal reaction force between the objects during impact time, \( dt \).

The collision of two objects can be divided into two separate phases, deformation and restoration. The deformation phase exists from the moment of contact until the two objects attain a common velocity. The restoration phase exists from the end of the deformation phases until the objects are separated. The ratio of these two impulses is defined as the coefficient of restitution, \( e \).

\[
mv + F\Delta t = mv \\
\Sigma mv = \Sigma mv
\]

For object b:

\[
v_a' > v_b' \quad \text{and} \quad \vec{U} \quad \text{and} \quad v_a < v_b
\]

\[
m_a v_a' + m_b v_b' = m_a v_a + m_b v_b
\]

For object b:

\[
v_b' \leftarrow -R \oplus U \\
U \leftarrow -P \oplus v_b
\]

\[
m_b v_b' = -\int Rdt + m_b U \\
m_b U = -\int Pdt + m_b v_b
\]

\[
e = \frac{\int Rdt}{\int Pdt} = \frac{m_b U - m_b v_b'}{m_b v_b - m_b U}
\]

Similar, for object a:
Therefore the coefficient of restitution, \( e = \sum \frac{\int Rdt}{\int Pdt} = \frac{m_a v_a' - m_a U}{m_a U - m_a v_a} \) (6)

The ratio is a measure of the elastic/elastic properties of the particles and must be determined experimentally. For perfectly elastic impact, the coefficient of restitution, \( e=1 \) and the kinetic energy of the objects is conserved. For perfectly plastic impact, the coefficient of restitution, \( e=0 \) and the objects move together at the same velocity after the impact, which corresponds to the maximum loss of kinetic energy. In reality, the coefficient of restitution for all real objects falls between \( 0 < e < 1 \). The coefficient of restitution is dependent upon impact velocity, sizes, shapes, and temperatures causing variations.

**Results and Discussion**

The data for the five trials for the aluminum and steel at 30\(^o\) and 50\(^o\) was recorded and can be seen below in Table 1 and Table 2 respectively.

For the 30\(^o\) elevation of the cylinder, the distance traveled by the aluminum block varied from 5.84 inches to 9.34 inches with an average of 7.34 inches. At the 30\(^o\) elevation of the cylinder, the resulting angle after impact ranged from 10\(^o\) to 12\(^o\) with an average of 10.7\(^o\). For the 50\(^o\) elevation of the cylinder, the average distance traveled by the aluminum block was around 18 inches. The resulting angle after impact ranged from 16\(^o\) to 18\(^o\) with an average of 16\(^o\).

**Table 1 - Aluminum Block**

<table>
<thead>
<tr>
<th>Trial</th>
<th>30(^o)</th>
<th>50(^o)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distance Traveled (in)</td>
<td>Resulting Angle (degree)</td>
</tr>
<tr>
<td>1</td>
<td>9.34</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>5.84</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>6.19</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>7.94</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>7.38</td>
<td>10.5</td>
</tr>
<tr>
<td>Average:</td>
<td>7.34</td>
<td>10.7</td>
</tr>
</tbody>
</table>

For the 30\(^o\) elevation of the cylinder, the distance traveled by the steel block varied from 2.781 inches to 3.125 inches with an average of 2.931 inches. At the 30\(^o\) elevation of the cylinder, the resulting angle after impact ranged from 1.0\(^o\) to 2.5\(^o\) with an average of 1.6\(^o\). For the 50\(^o\) elevation of the cylinder, the distance traveled by the block varied from 7.625 inches to 8.22 inches, with an average of 7.85 inches. The resulting angle after impact was equivalent to 0\(^o\) for all trials.
Table 2 - Steel Block

<table>
<thead>
<tr>
<th>Trial</th>
<th>Distance Traveled (in)</th>
<th>Resulting Angle (degree)</th>
<th>Distance Traveled (in)</th>
<th>Resulting Angle (degree)</th>
</tr>
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<tr>
<td>1</td>
<td>3.125</td>
<td>2.5</td>
<td>7.81</td>
<td>0</td>
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<tr>
<td>2</td>
<td>2.813</td>
<td>1.0</td>
<td>7.69</td>
<td>0</td>
</tr>
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<td>3</td>
<td>2.781</td>
<td>2.0</td>
<td>7.625</td>
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</tr>
<tr>
<td>4</td>
<td>2.938</td>
<td>1.5</td>
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<tr>
<td>Average:</td>
<td>2.931</td>
<td>1.6</td>
<td>7.85</td>
<td>0</td>
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</tbody>
</table>

From the data recorded the velocity before and after impact for the cylinder was calculated for each trial. The velocity before impact for the cylinder was calculated using the work/energy method. The velocities were 2.47 ft/sec for the 30° angle and 4.04 ft/sec for the 50° angle. In contrast the impulse/momentum approach was used to determine the final velocity after impact for the cylinder. For the aluminum and steel impact at a 30° elevation, the velocity of the cylinder ranged from 0.77 ft/sec to 1.124 ft/sec, with an average of 0.967 ft/sec. For the 50° elevation, the final velocity of the cylinder was 1.672 ft/sec. For the steel on steel impact, the final velocity of the cylinder for the 30° elevation ranged from -0.265 ft/sec to -0.111 ft/sec with an average of -0.179 ft/sec. For the 50° elevation, the final velocity of the cylinder ranged from -0.402 ft/sec to -0.239 ft/sec with an average of -0.301 ft/sec. Negative velocities is in reference to the direction traveled.

The impulse/momentum approach was used to calculate the initial velocity of the aluminum block right after impact as well. For the 30° elevation, the velocity after impact for the aluminum block ranged from 3.280 ft/sec to 4.147 ft/sec, with an average of 3.675 ft/sec. For the 50° elevation, the velocity of the block after impact was 5.76 ft/sec. Similarly, the velocity for the steel block after impact was also calculated. For the 30° elevation, the initial velocity of the block ranged from 2.168 ft/sec to 2.298 ft/sec, with an average of 2.226 ft/sec. For the 50° elevation, the velocity of the block after impact ranged from 3.590 ft/sec to 3.727 ft/sec, with an average of 3.643 ft/sec.

For comparison, the theoretical velocities for the cylinder and block after impact were determined using impulse/momentum with coefficient of restitution $e=1$. The average theoretical velocity for the cylinder after impacting the aluminum block was 1.302 ft/sec and 1.686 ft/sec for the 30° and 50° deflection respectively. For the cylinder impacting the steel block, the average theoretical velocity for the cylinder after impact was -0.210 ft/sec and -0.350 ft/sec. The average theoretical velocity for the aluminum block after impact was 3.502 ft/sec and 5.720 ft/sec for the 30° and 50° deflection respectively. The average theoretical velocity for the steel block after impact was 2.250 ft/sec and 3.680 ft/sec. Percentage error between theoretical and experimental velocities for the cylinder and block after impact was also analyzed. The percent error for velocity of the cylinder after impacting the aluminum block was 6.262% and 0.811% for the 30° and 50° deflection of the pendulum respectively. For the cylinder impacting the steel block, the percent error was 14.91% and 14.0% for the 30° and 50° deflection respectively.
percent error for the aluminum block after impact was 4.942% and 0.633% for the 30° and 50° deflection respectively. The percent error for the steel block after impact was 1.605% and 1.00% for the 30° and 50°. From analysis, velocity was determined to be directly related to mass, the heavier the mass the lower the velocity, which was verified by the data. For the 30° deflection the velocity for the aluminum block was higher than the velocity of steel. Similarly, for the 50° deflection, the velocity for the aluminum was also higher, 5.756 ft/sec versus 3.643 ft/sec.

In addition, the experimental coefficient of restitution and coefficient of friction were calculated and compared to the theoretical values. For this experiment, the theoretical coefficient of restitution was assumed to be 1 due to perfectly rigid body collisions. For the collision between aluminum block and steel cylinder at 30°, the coefficient of restitution varied from 0.872 to 1.182 with an average of 1.090. At 50° elevation, the coefficient of restitution was 1.012. The percent error between the experimental and theoretical coefficient of restitution was 9.00% and 1.20% for the 30° and 50° deflection. Similarly, the coefficient of restitution between steel block and steel cylinder was calculated. For the 30° elevation, the experimental coefficient varied from 0.922 to 1.037 with an average of 0.973. For the 50° elevation the coefficient of restitution varied from 0.957 to 1.023 with an average of 0.977. The percent error was 2.70% and 9.770% for the 30° and 50° deflection. The experimental coefficient of friction for the aluminum block and aluminum runway at the 30° elevation ranged from 0.245 to 0.391, with an average of 0.311. For the 50° elevation, the coefficient of friction was 0.339. Using 0.343 as the theoretical coefficient of friction for aluminum on aluminum, the percent error for the experimental data was calculated to be 9.33% and 1.20% for the 30° and 50° deflection. For the steel sliding across the aluminum runway, the coefficient of friction ranged from 0.302 to 0.339 with an average of 0.322 for the 30° elevation. For the 50° elevation, the coefficient of friction ranged from 0.307 to 0.331 with an average of 0.321. Percent error for COF was 2.222% and 1.905% for the 30° and 50° deflection. Theoretical and experimental calculations along with percent error are displayed below in Table 3-6.

<table>
<thead>
<tr>
<th>Trial</th>
<th>COF</th>
<th>V_a (work/en)</th>
<th>V_bf(imp/mom)</th>
<th>V_a</th>
<th>V_bf</th>
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<tr>
<td>1</td>
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<td>4.147</td>
<td>0.768</td>
<td>3.502</td>
<td>1.032</td>
</tr>
<tr>
<td>2</td>
<td>0.391</td>
<td>3.280</td>
<td>1.124</td>
<td>3.502</td>
<td>1.032</td>
</tr>
<tr>
<td>3</td>
<td>0.369</td>
<td>3.375</td>
<td>1.085</td>
<td>3.502</td>
<td>1.032</td>
</tr>
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<td>4</td>
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<td>0.901</td>
<td>3.502</td>
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<td>5</td>
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<td>0.958</td>
<td>3.502</td>
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<td>3.675</td>
<td>0.967</td>
<td>3.502</td>
<td>1.032</td>
</tr>
</tbody>
</table>

Table 3 - Aluminum Block - 30°

Percent Error for V_a (%): 4.942  Theor of e =1
Percent Error for V_bf (%): 6.262  Theor of COF = 0.343
Percent Error for e (%): 9.00  Percent error for e and COF are based on the average value
Percent Error for COF (%): 9.33
### Table 4 - Steel Block - 30°

<table>
<thead>
<tr>
<th>Trial</th>
<th>e</th>
<th>COF</th>
<th>$V_a$ (work/en)</th>
<th>$V_{bf}$ (imp/mom)</th>
<th>$V_a$</th>
<th>$V_{bf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.037</td>
<td>0.302</td>
<td>2.298</td>
<td>-0.265</td>
<td>2.250</td>
<td>-0.210</td>
</tr>
<tr>
<td>2</td>
<td>0.933</td>
<td>0.335</td>
<td>2.180</td>
<td>-0.125</td>
<td>2.250</td>
<td>-0.210</td>
</tr>
<tr>
<td>3</td>
<td>0.922</td>
<td>0.339</td>
<td>2.168</td>
<td>-0.111</td>
<td>2.250</td>
<td>-0.210</td>
</tr>
<tr>
<td>4</td>
<td>0.975</td>
<td>0.321</td>
<td>2.228</td>
<td>-0.182</td>
<td>2.250</td>
<td>-0.210</td>
</tr>
<tr>
<td>5</td>
<td>0.996</td>
<td>0.314</td>
<td>2.252</td>
<td>-0.210</td>
<td>2.250</td>
<td>-0.210</td>
</tr>
<tr>
<td>Average</td>
<td>0.973</td>
<td>0.322</td>
<td>2.226</td>
<td>-0.179</td>
<td>2.250</td>
<td>-0.210</td>
</tr>
</tbody>
</table>

Percent Error for $V_a$ (%): 1.065
Percent Error for $V_{bf}$ (%): 14.910
Percent Error for e (%): 2.7
Percent Error for COF (%): 2.222

Theo of $e = 1$
Theo of COF = 0.315

### Table 5 - Aluminum Block - 50°

<table>
<thead>
<tr>
<th>Trial</th>
<th>e</th>
<th>COF</th>
<th>$V_a$ (work/en)</th>
<th>$V_{bf}$ (imp/mom)</th>
<th>$V_a$</th>
<th>$V_{bf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.012</td>
<td>0.339</td>
<td>5.756</td>
<td>1.672</td>
<td>5.720</td>
<td>1.686</td>
</tr>
<tr>
<td>2</td>
<td>1.012</td>
<td>0.339</td>
<td>5.756</td>
<td>1.672</td>
<td>5.720</td>
<td>1.686</td>
</tr>
<tr>
<td>3</td>
<td>1.012</td>
<td>0.339</td>
<td>5.756</td>
<td>1.672</td>
<td>5.720</td>
<td>1.686</td>
</tr>
<tr>
<td>4</td>
<td>1.012</td>
<td>0.339</td>
<td>5.756</td>
<td>1.672</td>
<td>5.720</td>
<td>1.686</td>
</tr>
<tr>
<td>5</td>
<td>1.012</td>
<td>0.339</td>
<td>5.756</td>
<td>1.672</td>
<td>5.720</td>
<td>1.686</td>
</tr>
<tr>
<td>Average</td>
<td>1.012</td>
<td>0.339</td>
<td>5.756</td>
<td>1.672</td>
<td>5.720</td>
<td>1.686</td>
</tr>
</tbody>
</table>

Percent Error for $V_a$ (%): 0.633
Percent Error for $V_{bf}$ (%): 0.811
Percent Error for e (%): 1.200
Percent Error for COF (%): 1.20

Theo of $e = 1$
Theo of COF = 0.343

Percent error for $e$ and COF are based on the average value.
The theoretical velocity of the cylinder after impact was also calculated using the resulting angle measured from the apparatus. As shown in tables 7 and 8 below, the theoretical velocities of the cylinder varied from 0.000 to 1.334 ft/sec. These theoretical values were determined using the work energy approach which was based on gravity and the height of the cylinder after impact. The percent error was calculated for each block and angle, which resulted with an error from 8.277% to 234%. The discrepancies with the data can mainly be attributed to the accuracy of the measuring the resulting angle. For impacts (aluminum on aluminum), the resulting angle was large and more easily read, yielding more accurate results as shown in table 7 below. However, for the aluminum on steel collision, the resulting angle was very small and difficult to read. This resulted in inaccurate values, as can be shown in table 8 below.

<table>
<thead>
<tr>
<th>Trial</th>
<th>e</th>
<th>COF</th>
<th>( V_a ) (work/en)</th>
<th>( V_{bf} ) (imp/mom)</th>
<th>( V_a )</th>
<th>( V_{bf} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.968</td>
<td>0.324</td>
<td>3.627</td>
<td>-0.282</td>
<td>3.680</td>
<td>-0.350</td>
</tr>
<tr>
<td>2</td>
<td>0.957</td>
<td>0.328</td>
<td>3.605</td>
<td>-0.256</td>
<td>3.680</td>
<td>-0.350</td>
</tr>
<tr>
<td>3</td>
<td>0.949</td>
<td>0.331</td>
<td>3.590</td>
<td>-0.239</td>
<td>3.680</td>
<td>-0.350</td>
</tr>
<tr>
<td>4</td>
<td>0.988</td>
<td>0.318</td>
<td>3.663</td>
<td>-0.325</td>
<td>3.680</td>
<td>-0.350</td>
</tr>
<tr>
<td>5</td>
<td>1.023</td>
<td>0.307</td>
<td>3.727</td>
<td>-0.402</td>
<td>3.680</td>
<td>-0.350</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>0.977</strong></td>
<td><strong>0.321</strong></td>
<td><strong>3.643</strong></td>
<td><strong>-0.301</strong></td>
<td><strong>3.680</strong></td>
<td><strong>-0.350</strong></td>
</tr>
</tbody>
</table>

**Percent Error for \( V_a \) (\%): 1.00**
| Theo of \( e = 1 \) |

**Percent Error for \( V_{bf} \) (\%): 14.0**
| Theo of COF = 0.315 |

**Percent Error for \( e \) (\%): 9.770**
| Percent error for \( e \) and COF are based on the average value |

**Percent Error for COF (\%): 1.905**

Table 7 - Velocity of Cylinder After Impact (Aluminum Block)

<table>
<thead>
<tr>
<th>Trail</th>
<th>30° elevation (ft/sec)</th>
<th>50° elevation (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental</td>
<td>Theoretical</td>
</tr>
<tr>
<td>1</td>
<td>0.768</td>
<td>0.918</td>
</tr>
<tr>
<td>2</td>
<td>1.124</td>
<td>0.835</td>
</tr>
<tr>
<td>3</td>
<td>1.085</td>
<td>0.835</td>
</tr>
<tr>
<td>4</td>
<td>0.901</td>
<td>1.002</td>
</tr>
<tr>
<td>5</td>
<td>0.958</td>
<td>0.877</td>
</tr>
<tr>
<td>6</td>
<td>0.967</td>
<td>0.893</td>
</tr>
<tr>
<td>7</td>
<td>0.768</td>
<td>0.918</td>
</tr>
<tr>
<td>8</td>
<td>1.124</td>
<td>0.835</td>
</tr>
<tr>
<td>9</td>
<td>1.085</td>
<td>0.835</td>
</tr>
<tr>
<td>10</td>
<td>0.901</td>
<td>1.002</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>0.958</strong></td>
<td><strong>0.877</strong></td>
</tr>
<tr>
<td><strong>Percent Error</strong></td>
<td><strong>8.277 %</strong></td>
<td><strong>18.075 %</strong></td>
</tr>
</tbody>
</table>
Table 8 - Velocity of Cylinder After Impact (Steel Block)

<table>
<thead>
<tr>
<th>Trail</th>
<th>30° elevation (ft/sec)</th>
<th>50° elevation (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental</td>
<td>Theoretical</td>
</tr>
<tr>
<td>1</td>
<td>-0.265</td>
<td>0.209</td>
</tr>
<tr>
<td>2</td>
<td>-0.125</td>
<td>0.084</td>
</tr>
<tr>
<td>3</td>
<td>-0.111</td>
<td>0.167</td>
</tr>
<tr>
<td>4</td>
<td>-0.182</td>
<td>0.125</td>
</tr>
<tr>
<td>5</td>
<td>-0.210</td>
<td>0.084</td>
</tr>
<tr>
<td>6</td>
<td>-0.179</td>
<td>0.134</td>
</tr>
<tr>
<td>7</td>
<td>-0.265</td>
<td>0.209</td>
</tr>
<tr>
<td>8</td>
<td>-0.125</td>
<td>0.084</td>
</tr>
<tr>
<td>9</td>
<td>-0.111</td>
<td>0.167</td>
</tr>
<tr>
<td>10</td>
<td>-0.182</td>
<td>0.125</td>
</tr>
<tr>
<td>Average</td>
<td>-0.210</td>
<td>0.084</td>
</tr>
</tbody>
</table>

Percent Error: 234 % 100 %

Note: Percent Errors were calculated using average values

Discrepancies between theoretical and experimental velocities after impact can be attributed to the assumption that the coefficient of restitution was equivalent to 1. From analysis of the experimental data, the coefficient of restitution was shown to be not exactly 1. This can be attributed to the impact velocity, size, shape, and temperature causing variations. In addition, since the coefficient of restitution was not exactly 1, there was energy loss during impact between the cylinder and block. Another source of error can be attributed to oblique central impact of the cylinder and block. If the cylinder and block collide where the line of impact is not central then the collision is considered to be eccentric (oblique impact). Eccentric collisions reduce the distribution of energy which in turn reduces the velocities. Another contributing factor to the discrepancies in the calculated velocities was due to friction between the contact surfaces. Theoretically, the coefficient of friction between the block and surface should be the same for each trail. But as shown in our tabulated results, this coefficient of friction varied. This variation can be contributed to the different forces inflicted on the blocks caused by variations in speeds at which the system is initially run at. Secondly, surface defects played a role in the discrepancies with each trail when determining the coefficient of friction. The surface defects attributed to restricted motion on the block as it travels the length of the apparatus, implied that speed isn’t constant. But in order to determine the coefficient of friction, it is necessary to have a constant speed. As a result, the tabulated values needed to be calculated for each mass tested in order to reduce the percent error from these discrepancies.

Conclusion

Overall, this experiment demonstrated that more than one approach needs to be used to analyze collision of rigid bodies. In addition, this lab evaluated the velocities of different objects, with
different masses, set into motion at different deflection angles. An inverse relationship between mass and velocity was determined experimentally. Lastly, the coefficient of friction impacted all the calculations in one way or another. This internal force not only varied for each trial but was also a contributor to the discrepancies found when compared to the theoretical values.
Appendix: ASTM D 1894 Standard

(Only the first page of this Standard is attached here, the full document will be given to students as the handouts of this lab.)

Designation: D 1894 – 01

An American National Standard

Standard Test Method for Static and Kinetic Coefficients of Friction of Plastic Film and Sheeting

This standard is issued under the fixed designation D 1894; the number immediately following the designation indicates the year of
original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A
superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

This standard has been approved for use by agencies of the Department of Defense.

1. Scope

1.1 This test method covers determination of the coefficients of
static and sliding friction of plastic film and sheeting when
sliding over itself or other substances at specified test
conditions. The procedure permits the use of a stationary sled with
a moving plane, or a moving sled with a stationary plane. Both
methods yield the same coefficients of friction values for a
given sample.

Note 1—For the frictional characteristics of plastic film, partially
wrapped around a cylinder, or cup, see Test Method G 143 under
the jurisdiction of ASTM Subcommitte D 30.70.

1.2 Test data obtained by this test method is relevant and
appropriate for use in engineering design.

1.2.1 As an option to this test, coefficient of friction may be
run at temperatures other than 25°C by heating only the plane
while the sled is at ambient temperature.

1.3 The values stated in SI units are to be regarded as the
standard. The values given in parentheses are for information
only.

1.4 This standard does not purport to address all of the
safety concerns, if any, associated with its use. It is the
responsibility of the user of this standard to establish appropraiate
safety and health practices and determine the applicability of
regulatory limitations prior to use. For a specific precautionary
statement, see Note 7.

Note 2—This test method and ISO/DIS 8295–1994 are not technically
equivalent.

2. Referenced Documents

2.1 ASTM Standards:

D 618 Practice for Conditioning Plastics and Electrical
Insulating Materials for Testing

D 883 Terminology Relating to Plastics

D 3574 Test Methods for Flexible Cellular Materials—Slab,
Bonded, and Molded Urethane Foams

D 4000 Classification System for Specifying Plastic Materi-
als

E 691 Practice for Conducting an Interlaboratory Study to
Determine the Precision of a Test Method

G 143 Test Method for Measurement of Web/Roller Fric-
tion Characteristics

2.2 ISO/DIS Standards:

ISO/DIS 8295–1994

3. Terminology

3.1 Definitions:

3.1.1 friction, \( n \)—resistance to relative motion between two
bodies in contact.

3.1.1.1 coefficient of friction—the ratio of the force required

3.1.1.2 kinetic coefficient of friction—the ratio of the force

3.1.2 static coefficient of friction—the ratio of the force

3.2 Definitions of Terms Specific to This Standard:

3.2.1 slip—in plastic films, lubricity of two surfaces sliding
in contact with each other.

4. Significance and Use

4.1 Measurements of frictional properties may be made on a
film or sheeting specimen when sliding over itself or over
another substance. The coefficients of friction are related to the
slip properties of plastic films that are of wide interest in
packaging applications. These methods yield empirical data for
control purposes in film production. Correlation of test results
with actual performance can usually be established.

4.1.1 This test method includes testing at temperatures other
than 25°C by heating only the plane while the sled is at ambient

Note: A Summary of Changes section appears at the end of this standard.

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