INTEGRATED DESIGN AND ANALYSIS FOR A TREBUCHET USING A HIGH SPEED PHOTOGRAPHIC MEASUREMENT SYSTEM AND MATLAB

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1. ABSTRACT

We are expanding the scope of a freshman trebuchet design project to incorporate measurement and analysis for freshman and sophomores in the engineering curriculum. Students in our freshman level engineering design course have designed, built, and tested a trebuchet as part of the requirements for the class for many years. In the fall 2005 semester the operation of several trebuchets in the distance competition were recorded using a high speed photographic measurement system[1] on a laptop computer. These photographs were then analyzed by students in our freshman level programming class using Matlab to determine the arm and sling positions at several instants in time in their analysis co-ordinate system. The students in our sophomore level dynamics course also analyze the trebuchet as part of the requirements for that course. We plan to introduce a comparison between measured and predicted arm and sling positions in subsequent semesters of dynamics. In this paper we present the requirements for the trebuchet design, some of the results obtained from the measurement system, and the plan to expand the trebuchet analysis in the dynamics course.

2. INTRODUCTION

On 21 September, 2005, the EGR1710 class (graphics and design), a freshman level design course, conducted their trebuchet distance competition on the lawn north of the Seitz center. A CpE student collected data for several of the trebuchets using a Prosilica EC750 black and white scientific camera connected to a laptop via a firewire port. The camera power was provided from a 12V SLA battery connected to a fire-cable. Frames were captured in png format using Unibrain's Fire-I software and saved to the hard drive for subsequent analysis in Matlab.

A CD that included several shots from three different trebuchets in the distance competition was provided to students in the EGR1500 class (programming for engineers). The text for the class (Kaplan, 2004) includes a chapter on image processing and the analysis of the trebuchet images was offered as a

[1] This project was funded by a Lilly faculty development grant for the 2005-2006 academic year.

project in that section of the course. Several of the student groups in the programming class were also taking the design class, and so interest in the analysis of the trebuchet images was high.

Students in EM2020, a sophomore dynamics class, also analyze the trebuchet. We plan to introduce a comparison between the measured and theoretical results as a project for students in that class.

3. EGR1710 DESIGN PROJECT

The first team project assigned in EGR1710 is the design and construction of a light weight trebuchet. The project objectives are to follow the design process presented in class, work effectively as a team, practice presenting a design, and win the distance competition. Equal 20 pound weights were provided and teams were permitted no more than an eight foot height above the ground for this weight's starting position, and no more than fifty dollars for all materials, including donated materials which would have a value assigned by the course instructor.

Documentation for the first two steps of the design process, problem identification and preliminary ideas, was due within the first week of the assignment. This documentation included a problem description, sketches of at least three preliminary ideas, and a description of each idea. Documentation for the next three steps, refinement, analysis, and decision, was due the following week. This documentation included a matrix of pros and cons for each preliminary idea that included such issues as cost, reliability, manufacturability, safety, and so on, along with the rationale used to select the final design, and drawings of the final design. The final step of the design process, implementation, was due the next week. The documentation for this step included a parts list and class presentation. Students fabricated their trebuchets under the direction of the course instructor, and the distance competition took place at the end of the week. All systems were allowed several practice runs to adjust system components for optimum distance before completing three official distance trials. All systems were required to complete at least one trial for the team to receive credit for this portion of the project.

4. EGR1500 MEASUREMENT PROJECT

The first frame captured for one trebuchet is shown in Figure 1. The weight, arm, sling, and baseball are shown in the first frame where weight release was detected. Frames were taken with a .11 second spacing. Four frames spaced .22 seconds apart were analyzed in Matlab. Students were asked to create an overlay that included no less than four images of the arm and sling positions.

An example of a MATLAB script to create such an overlay is shown in Table 1. In the script, the overlay image was initially set to black, and then adjacent frames were compared to find the indices of pixels where the gray scale intensity had changed by more than a threshold value, T. These pixels represent that portion of the image that had moved between frames. The weight, arm, sling, and baseball positions are evident at .22 second intervals in Figure 2. Measured data was collected from trebuchet images in MATLAB using the ginput function. This data was then reduced from the pixel co-ordinates in MATLAB to the analysis co-ordinates using EXCEL. The angle that the arm makes with the vertical was also obtained from the EXCEL spreadsheet.

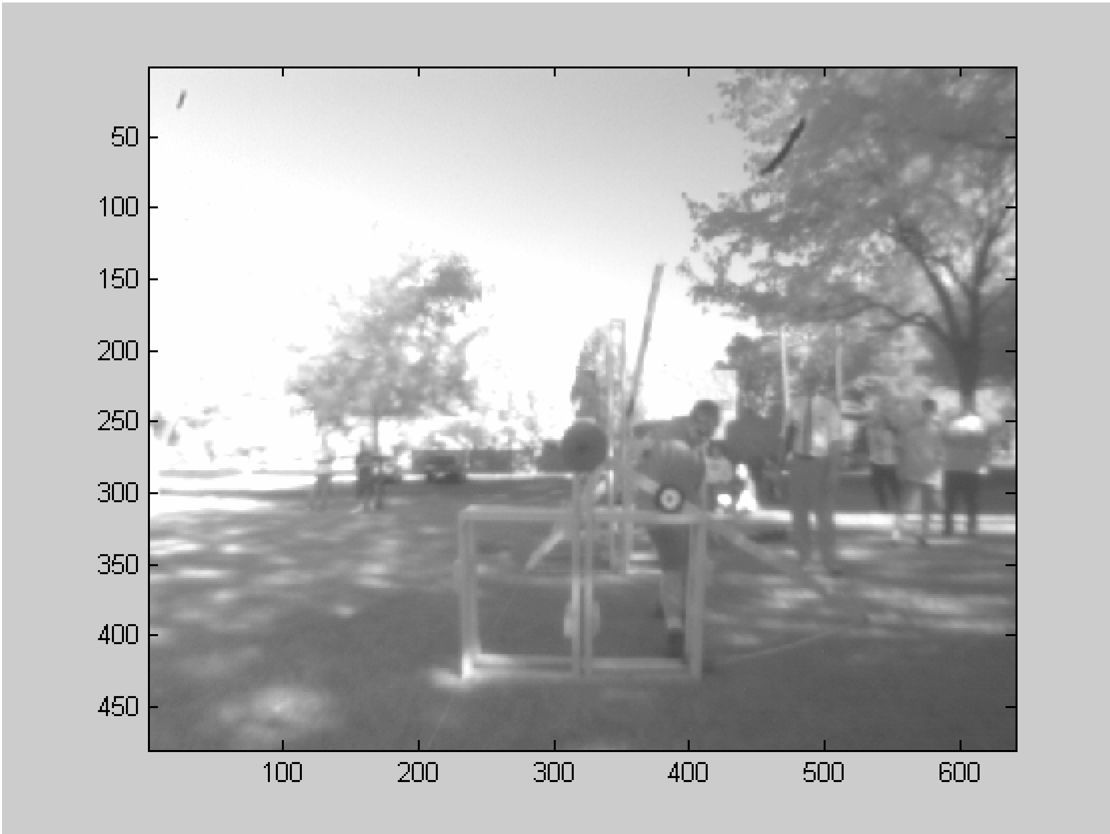

Figure 1: Initial photographic image – subsequent images spaced .11 seconds apart.

Table 1: Example MATLAB listing for overlay

```
%Script to overlay Trebuchet images, 10/24/05
T = 5;  %threshold for selection of pixels to overlay
a=imread('Inside Test_21_09_2005_15_52_44_171_001.PNG');
a_dbl = image2double(a);
c=imread('Inside Test_21_09_2005_15_52_44_390_003.PNG');
c_dbl = image2double(c);
e=imread('Inside Test_21_09_2005_15_52_44_609_005.PNG');
e_dbl = image2double(e);
g=imread('Inside Test_21_09_2005_15_52_44_828_007.PNG');
g_dbl = image2double(g);
overlay = zeros(size(a));
indx = abs(a_dbl-c_dbl)*255>T;
overlay(indx) = min(a_dbl(indx),c_dbl(indx));
indx = abs(c_dbl-e_dbl)*255>T;
overlay(indx) = max(c_dbl(indx),e_dbl(indx));
indx = abs(e_dbl-g_dbl)*255>T;
overlay(indx) = min(e_dbl(indx),g_dbl(indx));
figure(1), image(a);
colormap(gray(256));
brighten(.5);
figure(2), image(uint8(overlay*255))
colormap(gray);
title('.22 sec spacing')
```

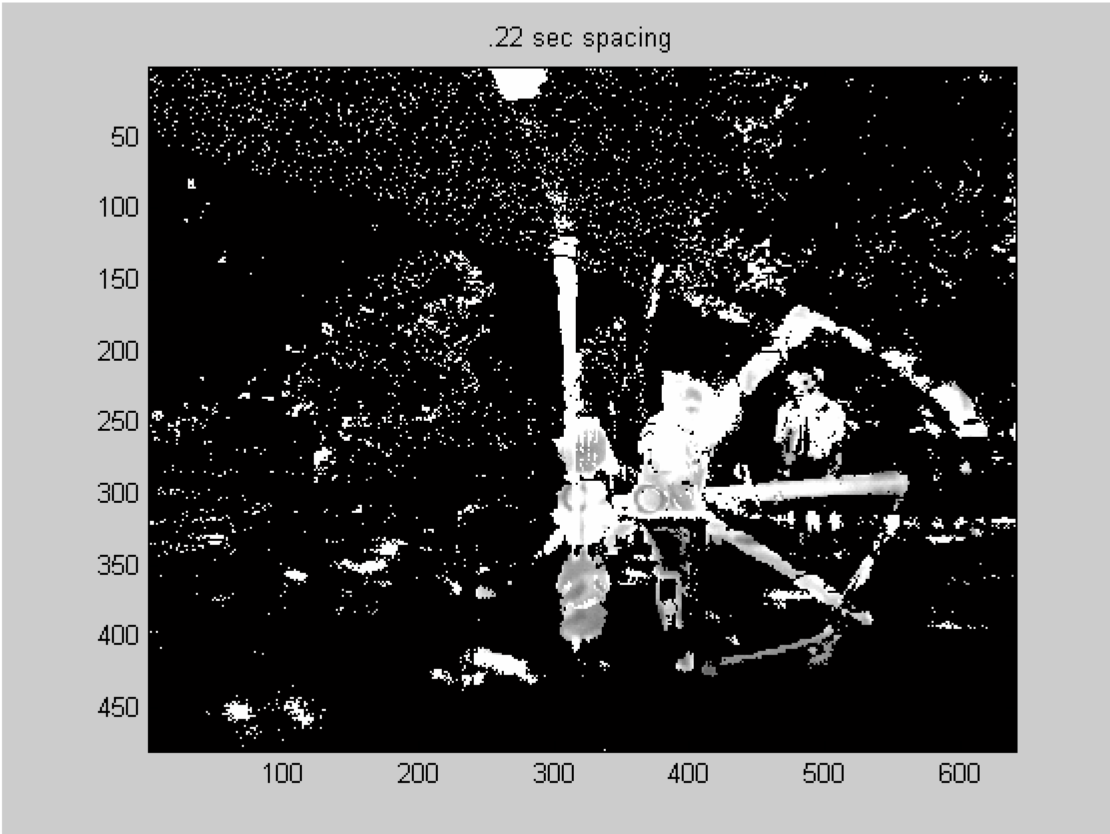

Figure 2: <u>Overlay of 4 images. The weight, arm, sling, and baseball positions can be seen at .22 second intervals.</u>

5. EM2020 ANALYSIS PROJECT

The experimental results obtained for each trebuchet will be compared against the theoretical results expected by students in EM2020 (dynamics), a fourth semester class in the mechanical and electrical engineering programs.

The geometry for the proposed model of the trebuchet arm is shown in Figure 3. Note that friction has been neglected and that the mass of the ball has been placed on the end of the arm. The proposed model is a one DOF system, and a straightforward extension of problems available in many undergraduate dynamics texts for constrained plane motion (Beer, *et al.*, 2004). A more complete model would include a force at the right end of the arm to model the tension in the sling, and a coupled model for the sling and ball. This model would be a one DOF model so long as the ball remains in contact with the ground. After the ball leaves the ground the model has two DOF. Such a model would be challenging for most undergraduates to analyze. Such an analysis, for a “hinged counterweight” trebuchet, is available in Jahsman (2005).

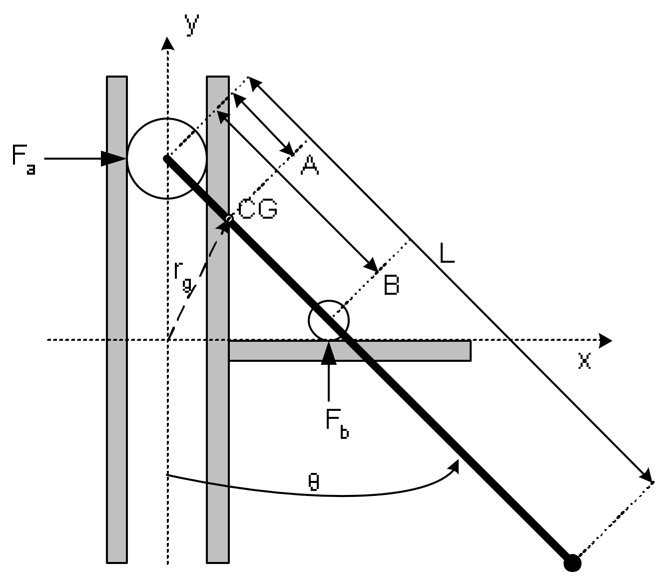

Figure 3: Trebuchet analysis geometry

Note that the constraint force, F_a, can be positive or negative. For translation, the EOM for the CG is given by

$$F_a \bar{i} - Mg\bar{j} + F_b \bar{j} = M\ddot{\bar{r}}_g \qquad (1)$$

where M is the total mass of the bar and where

$$\bar{r}_g = (B-A)\cos(q)\,\bar{j} + A\sin(q)\,\bar{i}. \qquad (2)$$

Solving for F_a and F_b in (1) leads to

$$\begin{aligned} F_a &= MA(\ddot{q}\cos(q) - (\dot{q})^2\sin(q)) \\ F_b &= Mg - M(B-A)(\ddot{q}\sin(q) + (\dot{q})^2\cos(q)). \end{aligned} \qquad (3)$$

Summing moments about the CG leads to

$$-AF_a\cos(q) + (B-A)F_b\sin(q) = I\ddot{q} \qquad (4)$$

where I is the moment of inertia. Substituting (3) into (4) leads to the nonlinear second order differential equation for θ

$$\ddot{q} = \frac{Mg(B-A)\sin(q) - MB(B-2A)(\dot{q})^2\sin(q)\cos(q)}{I + M(A^2 + B(B-2A)\sin^2(q))} \qquad (5)$$

which can be solved in MATLAB with the initial conditions $q(0) = q_0$ and $\dot{q}(0) = \dot{q}_0$.

The moment of inertia of the arm is straightforward to calculate using the parallel axis theorem:

$$I = (I_{weight} + M_{weight}A^2) + (I_{wheel} + M_{wheel}(B-A)^2) + (I_{ball} + M_{ball}(L-A)^2) + (I_{arm} + M_{arm}(\tfrac{L}{2} - A)^2)$$

where

$$I_{weight} = \tfrac{1}{2} M_{weight} r_{weight}^2$$
$$I_{wheel} = \tfrac{1}{2} M_{wheel} r_{wheel}^2$$
$$I_{ball} = \tfrac{2}{5} M_{ball} r_{ball}^2 \quad ,$$
$$I_{arm} = \tfrac{1}{12} M_{arm} (L^2 + b^2)$$

where b is the width of the arm, and where the center of gravity is

$$A = B\frac{M_{wheel}}{M} + \tfrac{L}{2}\frac{M_{arm}}{M} + L\frac{M_{ball}}{M} \quad .$$
$$M = M_{weight} + M_{wheel} + M_{ball} + M_{arm}$$

Measurements were taken on the trebuchet arm shown in Figure 1 and the center of gravity and moment of inertia were computed for the solution of equation 5 in Matlab. The code is shown below in Table 2.

Table 2: Matlab for the solution of equation 5.

```
%   One DOF Trebuchet model
global A B M I g
% definitions of constants
A=.2186;      % weight to CG (m)   .2310 with ball     .2186 without
B=.6096;     % weight to wheel (m)
M=12.44;    % mass (kg)            12.585 with ball    12.44 without
I=2.484;    % moment of inertia (kg-m^2)   2.856        2.484
g = 9.807;  % std. acceleration of gravity
ti = 0;
tf = .22;
x0=zeros(2,1);
x0(1)=60.23*pi/180; % initial position
x0(2)=30*pi/180;    % initial velocity
[t1,x1] = ode23('xprime_treb',[ti tf],x0);
%  extract theta at the final time
[r c] = size(x1);
th = x1(r,1)*180/pi

function xp=xprime_treb(t,x)
global A B M I g
xp=zeros(size(x));
s1=sin(x(1));
c1=cos(x(1));
xp(1)=x(2);
b=B*(B-2*A);
xp(2)=(M*g*(B-A)*s1-M*b*x(2)*x(2)*s1*c1)/(I+M*(A*A+b*s1*s1));
return
```

A comparison between the measured arm angles for the trebuchet of Figure 1 and the solution to equation 5 for two initial conditions is shown in Table 3

Table 3: Comparison between measured and theoretical results for the trebuchet in Figure 1.

Time (sec.)	Measured angle (deg.)	Computed angle for $\dot{q}_0$ = 0 deg./sec.	Computed angle for $\dot{q}_0$ = 30 deg./sec.
0	60.2	60.2	60.2
.22	96.9	73.1	79.7
.44	140.9	112.7	126.5
.66	183.4	181.8	201.0

Students in the dynamics class will be asked to reflect on the reasons for the differences in the measured and theoretical results in Table 3. These include:

1. It is difficult to identify the frame closest to the time when the weight was first released and the correct angular velocity for that frame.
2. In the more detailed model, the sling tension initially acts at a much more acute angle than the inertial force of the ball, and later acts to aid the arm rotation.

6. CONCLUSIONS

It has been noted recently (Halford, 2004) that the conventional "eat your spinach" approach to engineering education loses many engineering undergraduates to other majors before they ever reach a design class. We have developed an integrated design, measurement, and analysis project for freshmen and sophomores based on the trebuchet that engages undergraduates early in the engineering design process. The measurement of trebuchet arm and sling positions has been incorporated as a project in a freshman programming class. The analysis of the trebuchet arm will be conducted in a sophomore level dynamics class using a simplified one degree of freedom model. Students in the dynamics class will reflect on the modeling process to explain the differences between the measured and predicted arm positions.

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