USING MATHCAD® TO HELP VISUALIZE ELECTROMAGNETIC VECTOR FIELDS

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ABSTRACT

This paper describes an interactive Mathcad®* document which allows students in a first electrical engineering course in electromagnetic (EM) fields to quickly produce vector EM field plots on a computer. The document is intended to help students develop the ability to envision field patterns associated with physical sources and to reinforce the concept of a vector field. It is restricted to plots of the electric field in a plane perpendicular to two straight, parallel line charges having different linear charge density. Although restricted in geometry, it is flexible in use: it can be used as a synthesis tool with which students must find charge densities that produce a specified field at a specific point, as well as for analysis of the field produced by specific charges. An example of an assignment using this document is described. The advantages and compromises associated with using general mathematics software, such as Mathcad®, are compared to those of a dedicated program, such as a finite-element program. This document has been used for the first time in a course in Fall 2004, and is intended to be the first of several documents relating sources and field patterns. Thus, this attempt to nurture a link between the mathematics of EM fields and physical patterns is at an early stage. Student response to this initial effort will be presented.

1. INTRODUCTION

Electrical engineering (EE) students at Indiana Institute of Technology (Indiana Tech) work with discrete vectors in courses on statics and dynamics, and with vector fields in both physics and mathematics courses. The EE course on electromagnetics, EE 3500, however, is the first course dedicated to vector fields. Therefore, various exercises have been included in EE 3500 to reinforce the concept of a vector field, especially an electromagnetic field. A set of interactive computer exercises are being added, to complement the existing assignments which have required students to evaluate and sketch the field vectors along specified lines. The sections below describe the general features provided, details of the new Mathcad® document created; student reaction, and possible extensions. Images of vector plots from Mathcad® document, E_from_lineQ.mcd, are included.

2. FEATURES

In the planning phase, a number of desired features were identified to add value to a vector field visualization aid.   These desired features then determined the choice of program and the nature of the exercises developed in it. The paragraphs below describe some of the desired features and capabilities, and the resulting choices.

2.1 Use a general mathematic program.

Students often want to pigeon-hole course content, rather than see connections among various courses . To combat this tendency in some small way, this visualization aid uses Mathcad®, a general mathematics program that Indiana Tech EE students use in other courses, such as Signals and Systems, and Electrical Machines. Matlab®**, which is also used in a number of EE courses, would also have provided this feature. Use of any of the finite element method (FEM) programs, such as that available from Ansoft or Infolytica, might encourage compartmentalization: a student view that EM field problems require FEM, which is not used anywhere else in the EE program.

2.2 Provide quick results.

Since the intent is to help students develop intuition about electromagnetic field patterns, this visualization aid provides almost immediate change of the vector plot as source values or positions are changed. Although the plots provide correct values at each grid point, a course grid is used to ensure fast update of vector plots. The issue is not calculation speed, but immediacy of plot changes. For this feature, Mathcad®, with adjacent specifications and plots in an active document, is better suited than Matlab®, with basic command line structure; or FEM software, with post processing required for plot.

2.3 Display two-dimensional EM fields.

The fields are presented in the context of physical EM fields, not as mathematical abstractions. However, if point charge sources are used, with the magnitude of the electric field decreasing as distance squared, then arrow lengths in any vector plot decrease too quickly to spots to provide a good visual image, as shown in Figure 1. Walden and Christy (undated) addressed this problem by using two plots: a contour plot for electric field magnitude and a vector plot using unit vectors for direction. In the document described here, line charges (or charge "filaments" to emphasize the zero cross-sectional area) are used as sources, instead of point charges. The magnitudes of the resulting fields decrease linearly with distance, so vectors still have a reasonable image over some distance, as shown in Figure 2.   Further, the fields are fundamentally two-dimensional and therefore naturally presented in a two-dimensional plot.
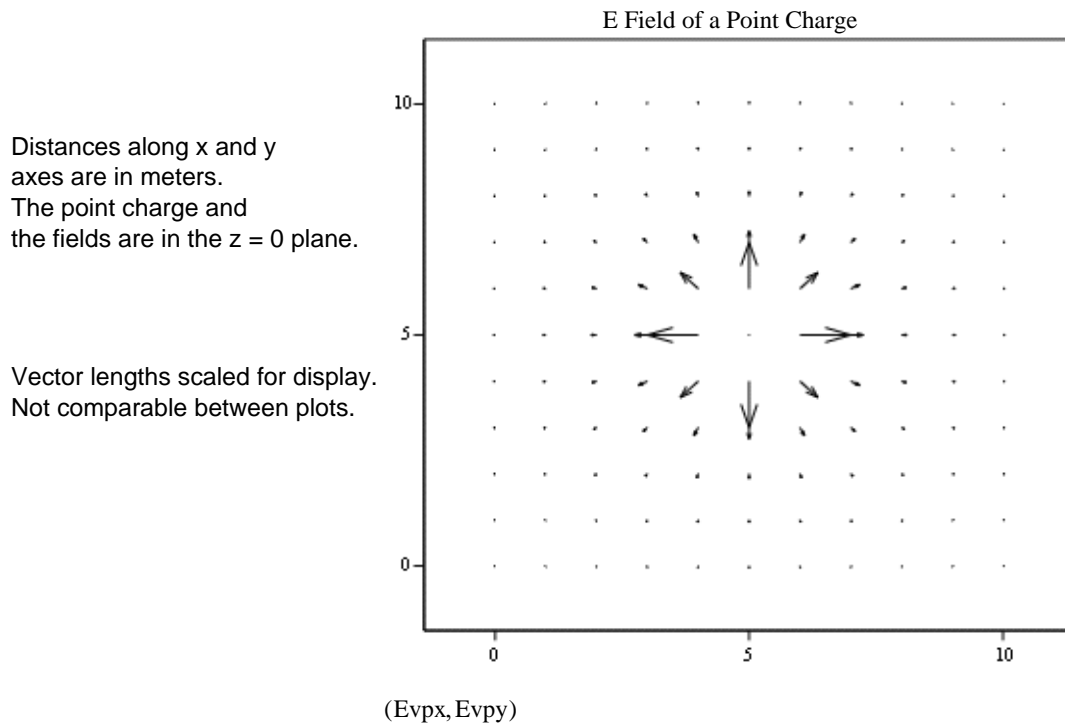
E Field of a Point Charge

Distances along x and y
axes are in meters.
The point charge and
the fields are in the z = 0 plane.

Vector lengths scaled for display.
Not comparable between plots.

(Evpx, Evpy)

Figure 1: Display of **E** field from a point charge, generated in Mathcad®.

E Field from Filament Charge

Distances along x and y
axes in meters.

The line or filament
is parallel to the z-axis.

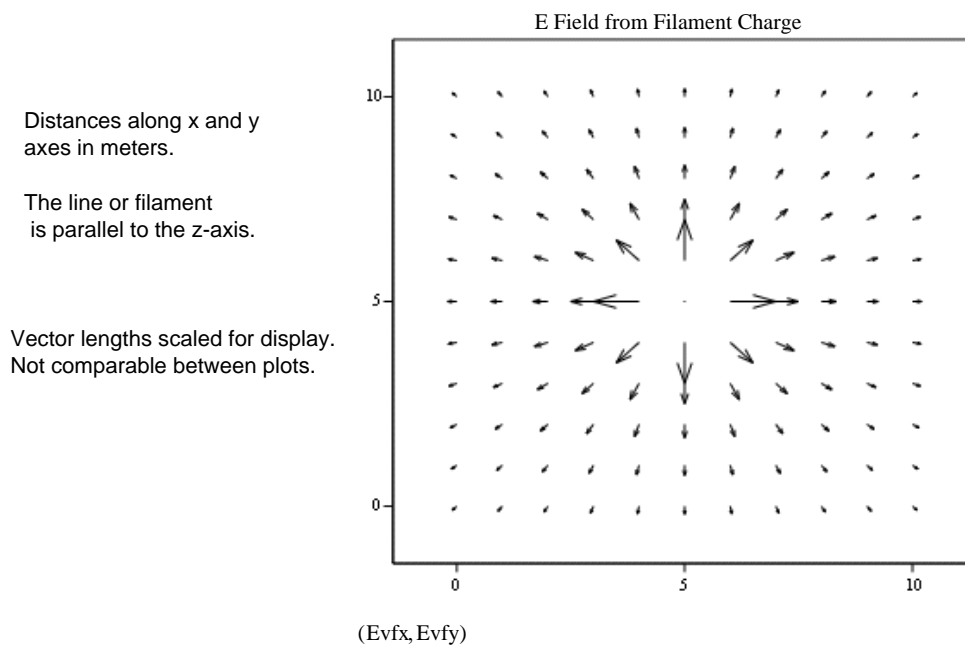Vector lengths scaled for display.
Not comparable between plots.

(Evfx, Evfy)

Figure 2: Display of **E** field from a charge filament, generated in Mathcad®.

2.4 Focus on vector field display.

Given that Mathcad® seems a good tool for the task, an obvious question is why bother to create a new or set of new documents? Why not use or adapt some available work, such as Visual Electromagnetics (Whites, 2000) or Schaum's Electronic Tutor (Edminister, 1995)? The answer is that these Mathcad® documents or electronic books cover a broad range of electromagnetic exercises, in contrast to the desired focus on vector field plots. The intent of this document is to help students develop the ability to envision fields by quickly providing vector field plots from sources. A new document was necessary to provide that focus.

3. DETAILS

In the Mathcad® document, E_from_lineQ.mcd, distributed to students, all parameters have been set to values that quickly produce vector plots with vectors having readily observable length and direction. However, all parameters can be changed, to encourage students to play "what if" games. Therefore, students are encouraged to save a separate copy for their own use.

The original document plots **E**-field vectors at 121 points on a 10 meter x 10 meter x-y grid from $(x,y) = (0,0)$ to (10 m, 10 m). Two charge filaments, parallel to the z-axis, intersect the x-y plane at (4 m, 5 m) and (6 m, 5 m). They are assigned equal negative linear charge densities. The associated vector plot is shown in Figure 3.

E Field from Two Filaments of Charge

Distances along x and y
axes are in meters.

Vector lengths scaled for display.
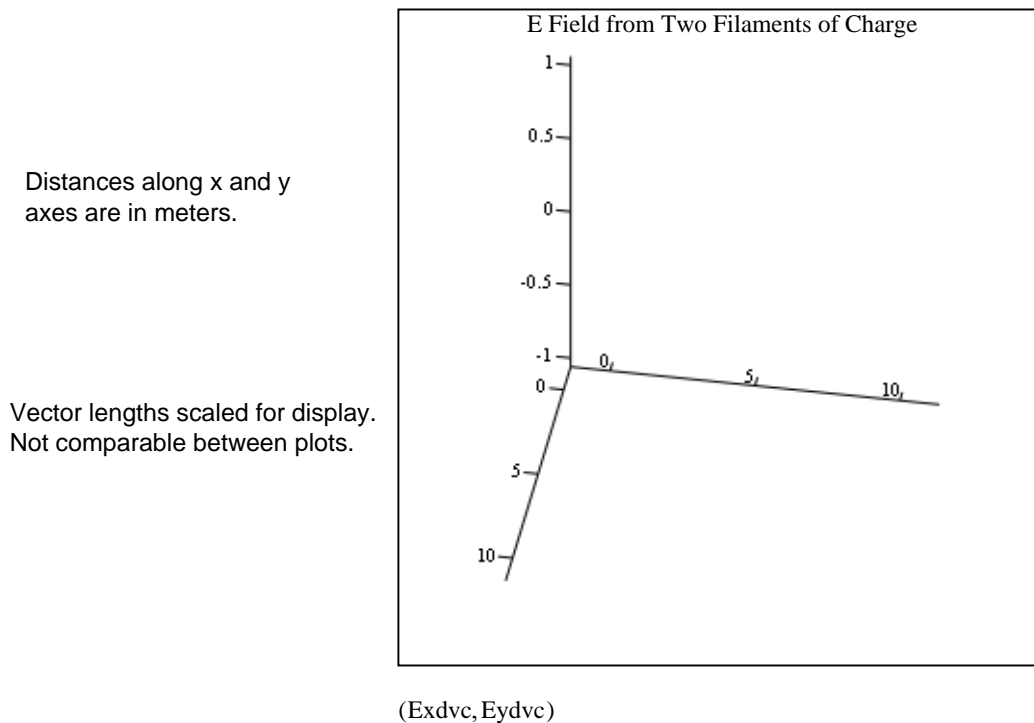Not comparable between plots.

$(Exdvc, Eydvc)$

Figure 3: Display of **E** field from two charge filament, generated in Mathcad®.

Assigned exercises for students can be of typical analysis type, in which the positions of the two filaments are specified, the two linear charge densities are specified, and a student must evaluate the field at a specified point. In addition, synthesis style problems are possible, in which a student must determine charge densities which produce a specified field at a specified point. Other combinations are obviously possible, including design problems in which a student must find filament positions (within the 10 m x 10 m space) and charge densities to produce a specified field at a specified point.

In any of these exercises, simultaneous use of the vector plot for guidance and Mathcad® calculations for numerical evaluations are required. The intention is that this simultaneous use begins to develop a student's intuition about vector field patterns.

4. STUDENT RESPONSE

To date, the E_from_lineQ.mcd document has been used for only one offering of EE 3500. The assigned problems were of the synthesis type described above, with fixed positions for the charge filaments. Each student was given a unique target field, although some field locations were repeated. The student then needed to determine the two linear charge densities which produced the specified field (x and y components) at the specified point. The assignment was given early in the semester, during review of basic field definitions in the course text (Hayt and Buck, 2004).

The average grade on the exercise was 78/100, with a sample standard deviation of 14/100 and a range from 50/100 to 85/100.

After the course was completed and grades submitted, students were asked to respond to the following questions, not anonymously. Too few students responded to permit any statistical conclusions, so only a general sense of the student response is provided after each question below.
1. Did you use the vector plots to guide your choice of linear charge density, or did you just guess, pure trial and error?
> The students felt that the vector plots were very helpful, but not absolutely necessary.
2. Did using the vector plotting help you envision vector fields, especially the distinction between x-y components of **E** and the x-y position at which **E** is evaluated (the field point)?
> The students preferred the active Mathcad®, but felt the text provided good plots, too.
3. Did you play with the Mathcad® document to see what happens?
> The students used the document a little beyond the assignment.
4. What did you like about using the document?
> The students thought the document easy to follow for the assignment and they liked to see the field patterns.
5. What needs improvement in the document?
> The students suggested little beyond adding a "readme" file.

5. EXTENSIONS

This Mathcad® document, E_from_lineQ.mcd, is intended as the first of a set being developed for EE 3500. The second of the set will present **H** field around a pair of wires. These documents may be enhanced by adding magnitude contour plots, as used by Walden and Christy (undated), to see if those plots add value for the students. As indicated in Section 3, these documents can be the bases for different types of exercises; in addition, adding more sources – charge filaments or wires – opens the opportunity for exercises that address uniqueness questions.

6. CONCLUSIONS

With two-dimensional fields and a coarse grid, Mathcad® can quickly produce vector plots which show the pattern of both field magnitude and direction with arrows. With this capability demonstrated, the important question remains: Can the vector plotting capability of a program like Mathcad® help EE students envision EM fields? At this point, there is not sufficient experience to reach a conclusive answer. The Mathcad® document will be extended and used in the future.

* Mathcad® is a registered trademark of Mathsoft, Inc.
** Matlab® is a registered trademark of The Math Works, Inc.

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