First Year Students' Understanding of Normal Distributions: A Preliminary Study of Previous Exposure, Self-Efficacy and Content Knowledge

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Calls to improve engineering education commonly emphasize the importance of students' ability to transfer, integrate, and apply knowledge in math and science to engineering contexts. In this study, I used items from existing instruments to examine the relationships of self-efficacy and previous learning with students' ability to transfer mathematical knowledge. A survey instrument gathered quantitative, baseline data from 112 students about (1) students' previous exposure to the normal curve, and (2) students' self-efficacy. In addition, students answered eight multiple-choice questions related to the concept of a normal distribution. Correlation tables revealed that two-thirds of the students' self-reported measures of self-efficacy scores were significantly correlated with their content knowledge ($r > .20$, $p < .05$). The hierarchical regression model revealed that self-efficacy was the only significant predictor of students' content knowledge; students' reports of previous exposure were non-significant. Findings suggest that assuming students' ability to transfer content knowledge simply based on prior course history is problematic.

Introduction

Calls to improve engineering education commonly emphasize the importance of students' ability to transfer, integrate, and apply knowledge in math and science to engineering contexts.[2-4] While a common definition of engineering may include "using math and science to solve problems," introductory engineering curricula may not always make clear the ways in which students are expected to transfer their math and science knowledge to solve engineering problems. Similarly, engaging in engineering design may not necessarily promote the learning of science and math concepts. This study is motivated by a need to understand how students do (or do not) transfer foundational content knowledge from math and science courses to engineering contexts.

Specifically, the transfer of mathematical concepts is the focus of this work. The emphasis on math (over science) is justified because math, specifically the practice of modeling, "drives the bulk of [working engineers'] sense-making activity, and defines professional expertise."[5] In addition to the use of math in the engineering workplace, students' perceptions of math may develop through their math classroom experiences to be at odds with the beliefs about mathematics held by teachers of mathematics and mathematicians, which can inhibit their achievement.[6] For example, Schoenfeld documented a false belief in students that "the mathematics learned in school has little or nothing to do with the real world."[6]

In addition to the need of math to practice engineering and the skewed perceptions of math developed within classroom settings, high school calculus achievement is a strong predictor of success in entry-level engineering courses such as calculus and physics.[7] In terms of experiencing design, K-12 engineering curricula may serve as a buffer to increase students' accessibility to studying engineering. However, despite many claims that engineering education helps improve math achievement, a recent study showed that after controlling for prior achievement and other differences in both teachers and students, students enrolled in pre-college curriculum showed significantly less gain on standardized math exams.[8]

This study is built on the assumption that engineering curriculum, when present in a K-12 setting, should increase math achievement of all students to increase their ability to participate in engineering, and that should be evident in a population of first year engineering students. In general, exposure to and attitudes about mathematics can act as a gateway for achievement in engineering curricula. This work provides baseline data that analyzes the relationship between students' views of mathematics in the form of self-efficacy, their exposure to math concepts in varied contexts, and their performance on mathematical content items.

Motivation

Additional rationale for studying mathematics and students' ability to transfer mathematics to engineering context is provided here. Research shows that inequities in math education in the U.S. disproportionally affect students of underrepresented minorities. While the 2009 National Assessment of Educational Progress (NAEP) reported nationwide improvements in national math scores between 2005 and 2009, the gaps between math achievement of black and Hispanic students compared to white students remain significant, shown in Figure 1.[1]

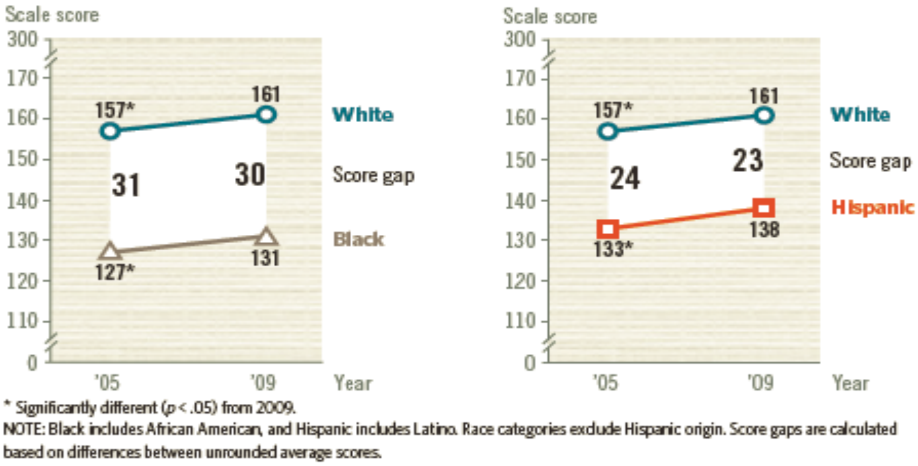

Figure 1. Twelfth-grade NAEP mathematics score gaps by selected racial/ethnic groups: 2005 and 2009.[1]

Students in lower income communities can be considered to be at a disadvantage to truly access engineering curriculum at rates comparable to their peers due to the documented achievement gap in math at the K-12 level. Closing this gap is crucial to increasing diversity in engineering because minority students, who are disproportionately of lower socioeconomic status (SES), remain underrepresented.

This issue of access is the larger context of recruiting and retaining a more diverse population into engineering that frames this line of inquiry. Many researchers have tried to provide strategies to increase diversity in engineering, but despite continuous efforts to improve the recruitment and retention of women and underrepresented minorities in engineering the numbers remain rather stable. A long term goal of looking at how students of varied beliefs about and exposure to math concepts transfer that knowledge to engineering contexts is one to increase the accessibility of engineering to a wider diversity of students. This larger motivation is beyond the scope of the data presented here, but remains a motivation for the researcher.

Research Question

This research provides a starting point for future research on learning how self-efficacy and the context of learning are related to students' ability to transfer mathematical knowledge. This line of inquiry has implications of informing K-12 engineering's potential to leverage engineering curriculum to increase students' math readiness and therefore access to engineering studies and drawing attention to the problematic nature of assuming students' content knowledge based on their previous course history. The three basic hypotheses that this research tested are as follows:

H1: Student's statistics self-efficacy scores will be significantly correlated with their content knowledge.
H2: Students' amount of previous exposure to the content will be significantly correlated with their content knowledge.
H3: Both self-efficacy and previous exposure to content knowledge will be predictive of variance in students' content knowledge.

Methods

The methodological decisions that guided this study are detailed in this section.

Instrument Development

To study how students transfer math knowledge from varied contexts, including engineering, a specific topic must be selected. For this research, the mathematical concept that was focused on is from statistics: the normal distribution. This is a key concept taught at both the high school and post-secondary level. The concept of a normal distribution is present in both the common core standards as well as the current standards in the state of Indiana. A recent study focusing on this criterion argues that while engineering problem solving commonly requires statistical thinking, few engineers posses the intuition about distributions and statistical modeling at an appropriate level.[9] In addition, this is a mathematical concept that has many practical applications across disciplines, which makes it an ideal selection for research related to the transfer of knowledge.

A preliminary instrument was developed to gather self-reported data from first year engineering students about the context in which they received exposure to the normal curve and their ability to answer questions about the normal curve. In addition to understanding how multiple contexts of learning may affect students' ability to answer content knowledge, another self-reported measure was collected: self-efficacy. This measure has been widely used in educational research since Bandura's work solidified the term.[10] Specifically, mathematics self-efficacy has been shown to be moderately correlated with mathematics performance.[11] The construct of self-efficacy was also included because research has revealed that mathematics self-efficacy weighs heavier in regression analysis than both mathematics performance and achievement variables when predicting students' selection of a math-related major.[11]

The assessment items can be seen in Appendix 1. Item 1 contained six Likert scale-rating questions to measure students' self-efficacy for several different statistical constructs. These items are based on the Mathematics Self-Efficacy Scale [12]. The content questions (2-6) are taken from the Statistics Concept Inventory. These items have been found to be valid and reliable measures of students' understanding of the normal distribution [13]. The next two items (7-8) were adopted from the course textbook [14]. The final item (9) is a statistics item released from the National Assessment of Educational Progress [15]. The final items (10-12) were designed by the researcher to gather information on how often and in what context students report being exposed to the natural curve and related concepts. Again, this was with the intention of gathering preliminary data related to how students' mathematical self-efficacy and the context of students' exposure to math content are associated with their transfer of that knowledge to engineering-like problems.

Validity and Reliability

Reliability testing was not performed on the assembled items as a comprehensive survey, but as explained, each item was either taken directly from or adopted from a valid and reliable existing instrument. In the development of the statistics concept inventory (SCI), experts were iteratively consulted to review items to ensure content validity. This provides evidence that the items selected for this study are adequate to measure the concept of interest, the normal distribution. Confirmatory factor analysis was not performed on the self efficacy items to confirm that they loaded on the different factors that were represented by the content questions, so future work would need to expand the rigor of this instrument in order to take account for this. Face validity was achieved by having the assessment reviewed by two different professors of the first year engineering course during its development. It would also be necessary for the expansion of this work to develop a way of improving the accuracy of students' previous exposure. This can be difficult as there is no guarantee of what is covered in classrooms despite the expectations or established standards.

Participants

A total of 125 students submitted the assignment, but 13 were removed because they did not provide consent for their anonymous responses to be used for future research. The researcher gained access to the submissions after IRB was approved in the last week of November 2012. Basic data analysis using SAS was done on the remaining 112 submissions. Some submissions were missing information, so the regression portion of the analysis used only 60 of the data points.

Data Collection

Data was collected by reviewing the submissions to an in-class assessment from one section of a first year engineering class at Purdue University. This assessment was developed by the researcher in alignment with the course instructor's desired topic of review of normal distribution, which was covered in class and on a homework assignment. This assessment was distributed to students using an electronic data system through Purdue University, Qualtrics as a part of their normal coursework. After the course instructor de-identified the data, it was provided to the researcher for analysis.

Results

The statistical package SAS was used to analyze the data from the submissions of the first-year engineering students. To begin, the multiple choice questions were coded as 0 (incorrect) or 1 (correct) so that a total score for the eight content questions (2-9) could be calculated. On average, students correctly answered 50% of the questions ($M = 3.98$, $SD = 1.72$). On the self-efficacy scale of 1-5, (5 indicated the highest possible level of confidence) the average reported confidence for statistical concepts was also calculated ($M = 3.85$, $SD = 0.70$). Correlations between the measures of self-efficacy and the content score for each participant are shown in the bottom row of Table 1. The students' self-efficacy for selecting the appropriate statistic and interpreting the mean of a data set were not significantly ($p < .05$) correlated to their total content score. The other four self-efficacy measures: interpret the standard deviation of a data set, decide if statistics are reported appropriately, use statistics in engineering problems, explain the normal curve in every day language were all significantly correlated with the students' final score for the content items. This addresses H1: Student's statistics self-efficacy scores will be significantly correlated with their content knowledge. This was true for four of the six self-efficacy items, but not for the first two as shown in Table 1.

Table 1. Correlations Between Youths' Mathematical Self-Efficacy and Content Score ($n = 112$)

Variables	1	2	3	4	5	6
1. Select Appropriate Statistic	--					
2. Interpret the Mean	.27**	--				
3. Interpret the SD	.40***	.39***	--			

4. Decide if Statistics are Reported Appropriately	.32***	.43***	.44***	--		
5. Use statistics in engineering problems	.28**	.30**	.37***	.67***	--	
6. Explain the Normal Curve In Every-day Language	.33***	.31***	.54***	.61***	.61***	--
7. Total Content Item Score	.07	.12	.28**	.24*	.21*	.20*

Note. *$^*p < .05$, $^{**}p < .01$, $^{***}p < .001$*

The items for exposure to the normal distribution were combined into frequency scores for exposure as the total number of classes that covered normal distribution content in high school and college. On average, students reported seeing the normal distribution in one of their high school classes ($M = 1.24$, $SD = 0.54$) and one of their college classes ($M = 1.21$, $SD = 0.49$). For high school exposure, the most frequently reported class for exposure to the normal curve was math (n = 51). For college, the most frequently reported class for exposure was engineering (n = 65) followed by math (n = 25).

Correlation values for the measures of total exposure to normal distribution content and total score on content items are provided in the bottom row of Table 2. Students' total score on the content items were not significantly ($p < .05$) correlated with their self-reports of exposure to the normal distribution in either high school or college. In addition to being non-significant, the correlations were slightly negative. This addresses H2: Students' amount of previous exposure to the content will be significantly correlated with their content knowledge. This hypothesis was rejected as neither reported exposure in high school or reported exposure in college was significantly correlated with students' content knowledge.

Table 2. Correlations Between Previous Exposure to Content and Content Score (n=112)

Variables	1	2	3
1. Total High School Exposure	--		
2. Total College Exposure	.64***	--	
3. Total Score	-0.18	-.10	--

Note. *$^*p<.05$, $^{**}p<.01$, $^{***}p<.001$*

A series of hierarchical regressions were used to examine the relationship between self-efficacy, previous exposure, and students' content knowledge. The initial model regressed students' content knowledge onto his/her self-efficacy and previous exposure in both high school and college. This model showed that self-efficacy is the only significant predictor of students' content knowledge ($b = .75$, $SE = .35$, $p < .05$, $\beta = .23$). Neither measure of previous exposure significantly contributed to prediction of variance in content knowledge. After these two predictors were removed, the final model containing only self-efficacy as a predictor explained a significant amount of the variation in students' content knowledge; $R^2 = 7.22\%$, $F(1, 60) = 4.59$, $p < .05$. Analyses tested the interaction between previous exposure and self-efficacy; no interactions were significant, so they were not included in the model. This addresses H3: Both self-efficacy and previous exposure to content knowledge will be predictive of variance in students' content knowledge. Results show that only self-efficacy was a significant predictor of variation in content knowledge.

Discussion

This preliminary work provided a starting point for future research on learning how self-efficacy and the context of learning affects students' ability to transfer mathematical knowledge with the intention of informing how engineering in K-12 can leverage engineering curriculum to increase math readiness and therefore access to engineering studies with students of varied SES. Numerous instruments exist to measure self-efficacy and conceptual understanding of statistics, which can provide valid and reliable items for work in this research area. However, the development of an instrument to tie these areas of research together with the intention of studying transfer of knowledge has been addressed uniquely by this study.

The self-efficacy data analysis revealed that students' reports of confidence for selecting appropriate statistics and interpreting the mean are not significantly correlated with their ability to answer content questions. This can be due to the wording of the self-efficacy questions—the discourse used in math learning affects responses to this type of item. The few number of self-efficacy items and the lack of confirmatory factor analysis needed to ensure that the predictive items are well aligned with the content questions also limit this research.

Next, the students' report of their exposure in either high school or college was negatively correlated with their ability to answer content questions, but not significantly at an alpha level of 0.05. This is a concerning finding because it suggests that students are not transferring knowledge that they have learned previously adequately to answer conceptual content questions. This data is also limited by the fact that it is self-reported data, so students may not report accurately. This certainly seems to be the case here; 47 of the students did not report exposure to the normal distribution in the class they were currently reviewing to see it on a test in! This supports literature that explains that without making discourse and connections very explicit, students fail to make connections intended by the curriculum being implemented.

Under the assumption of most educational systems that are hierarchical and cumulative, it is surprising that this data analysis revealed that self-efficacy was the only significant predictor of variation in content knowledge. This suggests that the current structure of many undergraduate programs can be problematic by focusing on math and science courses early in the degree and then assuming that students' knowledge gained there will be transferred to higher level engineering and engineering science courses. This assumption has indeed been documented in multiple schools, which could serve as a barrier to learning.[16]

It is also worthwhile to note that the topic of knowledge transfer itself is highly debated. In a comprehensive review of transfer literature, Barnett and Ceci synthesized a century's worth of research and reported, "there is little agreement in the scholarly community about the nature of transfer, the extent to which it occurs, and the nature of its underlying mechanisms."[17] Some argue that transfer is ubiquitous as an inherent mechanism of learning, while others claim it doesn't spontaneously occur. Perkins and Salomon point out that "no absolute line can be drawn between ordinary learning and transfer."[18] Established ways of measuring transfer often focus on transferring a concept from one context to another, analogous context. This phenomenon is of great interest for engineering education, but much additional work beyond what is presented here is needed to develop ways of measuring this with high levels of validity and reliability.

Recommendations

This research supports the need for further inquiry into the role that context of learning plays in the transfer of knowledge. Results further the concern of related previous research: in many curricula, while math concepts may be implicitly integrated, there are very few occasions where the mathematics concepts are explicitly integrated with the engineering activities intended for each lesson.[19] The assumption that students will retain and transfer mathematical concepts learned previously to engineering contexts with ease is worthy of skepticism. Discourse can also play a role with students' ability to make connections—as novices they may not recognize analogous situations as opportunities for transfer, so engineering educators should make an effort to highlight opportunities for transfer. The results presented here highlight that students do not always receive the intentions of the curriculum as expected. This work also suggests that math in K-12 isn't necessarily facilitating the skills expected of engineering students at the college level. The focus of the curriculum tends to be on obtaining measurements and organizing, gathering, and presenting data rather than solving for unknowns or creating mathematical models that cause students to make explicit connections between the math they are using and the physical phenomenon that it is describing in order to solve a real-world problem.[8] The development of these mathematical skills is critical to success in engineering, especially for students who are both underserved by K-12 in math education and underrepresented in engineering. Engineering educators need to continue to improve their methods of educating students in order to promote transfer of conceptual understanding beyond the context in which it was learned.

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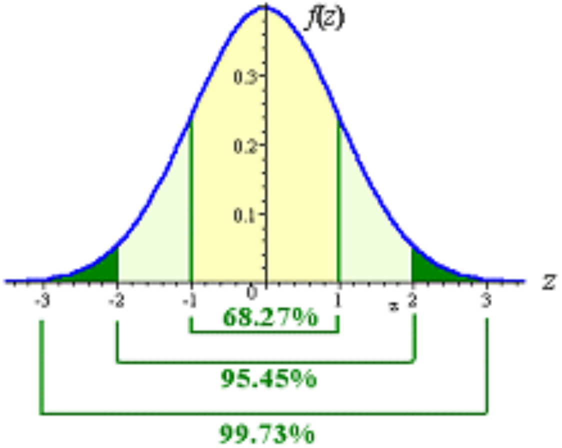
Appendix 1: Assessment Including Correct Answers (Bold)

The image below of a normal distribution is provided for your reference.



1. The following questions ask you to estimate your own statistics ability. On a scale of 1 to 5, how *confident* are you that you can perform each of the following statistical tasks?

	Not at All Confident				Very Confident
Select the appropriate statistic to describe a set of numbers	1	2	3	4	5
Interpret the mean	1	2	3	4	5
Interpret the standard deviation	1	2	3	4	5
Decide if statistics are used appropriately (e.g. in a newspaper article)	1	2	3	4	5
Use statistics to solve a real world engineering problem	1	2	3	4	5
Explain how to apply the normal distribution in every day language	1	2	3	4	5

2. A small object was weighed on the same scale separately by nine students in a science class. The weights (in grams) recorded by each students are shown below.

   6.2 6.0 6.0 15.3 6.1 6.3 6.2 6.15 6.2

   The students want to determine as accurately as they can the actual weight of this object. Of the following methods, which would you recommend they use?
   a. Use the most common number, which is 6.2.
   b. Use the 6.15 since it is the most accurate weighing.

- c. Add up the 9 numbers and divide by 9.
- **d. Throw out the 15.3, add up the other 8 numbers and divide by 8.**

3. For the following set of numbers, which measure will most accurately describe the central tendency?

3, 4, 5, 6, 6, 8, 10, 12, 19, 36, 83

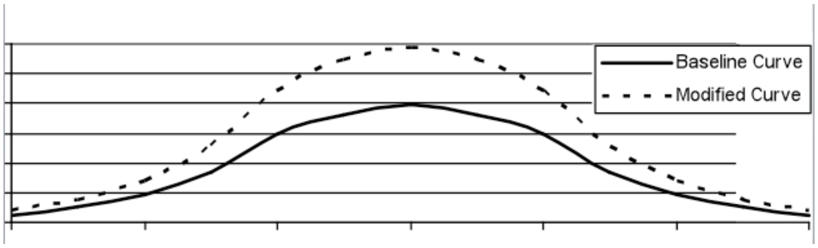
- a. Mean
- **b. Median**
- c. Mode
- d. Standard Deviation

4. Which statistic would you expect to have a normal distribution?
    - I. Height of women
    - II. Shoe size of men
    - III. Age in years of college freshman
    - **a. I & II**
    - b. II & III
    - c. I & III
    - d. All 3

5. You have a set of 30 numbers. The standard deviation from these numbers is reported as zero. You can be certain that:
    - a. Half of the numbers are above the mean
    - b. All of the numbers in the set are zero
    - **c. All of the numbers in the set are equal**
    - d. The numbers are evenly spaced on both sides of the mean

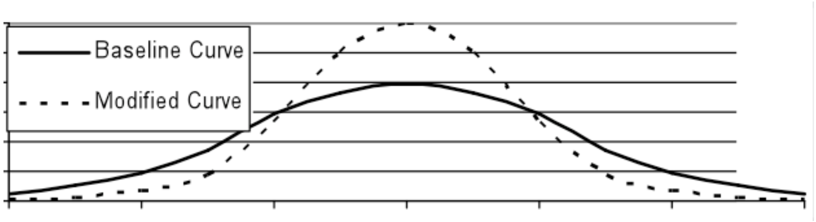
6. The mean height of American college men is 70 inches, with standard deviation 3 inches. The mean height of American college women is 65 inches, with standard deviation 4 inches. You conduct an experiment at your university measuring the height of 100 American men and 100 American women. Which result would *most* surprise you?
    - a. One man with height 79 inches
    - b. One woman with height 74 inches
    - c. The average height of women at your university is 68 inches
    - **d. The average height of men at your university is 73 inches**

7. Two experiments were conducted to measure and record the tensile strength of ligaments in artificial knees after being produced using artificial materials. Judging from the figure below, what changed from the baseline to the modified experiment?



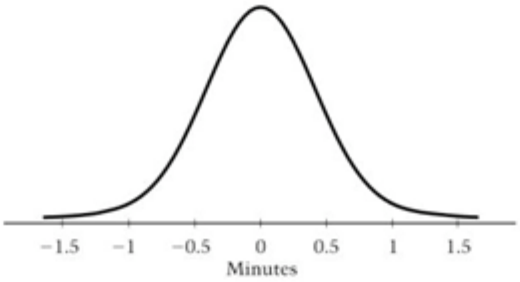
a. The modified experiment had a higher average tensile strength
b. The modified experiment had a higher number of samples
c. The baseline curve had a smaller standard deviation
d. The baseline curve had a smaller median tensile strength

8. A local newspaper contains an advertisement for slicing cucumbers for pickling. They present the following data with slice thickness as the independent variable from a traditional slicer (baseline) and their new slicer (modified). What conclusion in their advertisement would you believe?



a. The new slicer sliced more cucumbers than the old slicer
b. The new slicer produces thicker slices than the old slicer
c. The old slicer is less accurate than the new slicer
d. There is no difference between the two slicers

9. A clock manufacturer has found that the amount of time their clocks gain or lose per week is normally distributed with a mean of 0 minutes and a standard deviation of 0.5 minutes, as shown below.



In a random sample of 1,500 of their clocks, which of the following is closest to the expected number of clocks that would gain or lose more than 1 minute per week?

a. 15
b. 30
c. 50
d. 70
e. 90

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10. Where you exposed to the normal distribution **<u>before college</u>**? Circle all that apply:

e. Yes, in math class

f. Yes, in science class
g. Yes, in engineering class
h. Yes, but I don't remember where
i. No exposure
j. I don't remember
k. Other (please explain):____

11. Where you exposed to the normal distribution **<u>in college</u>**? Circle all that apply:
    l. Yes, in math class
    m. Yes, in science class
    n. Yes, in engineering class
    o. Yes, but I don't remember where
    p. No exposure
    q. I don't remember
    r. Other (please explain):____

12. In how many of your college classes have you covered the normal distribution?
    s. 0
    t. 1
    u. 2
    v. 3
    w. 4
    x. 5+