Finite Element Modules for Demonstrating Critical Concepts in Engineering Vibration Course

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Abstract
Machine components subjected to vibration can fail because of material fatigue. Vibration also causes more rapid wear of machine parts and creates excessive noise. It is beneficial for mechanical engineering students to understand vibration, analyze vibration and predict the behavior of vibrating systems.

This paper documents the development of a series of FE models for illustrating a variety of vibration phenomena, including the response to harmonic excitation and impulsive excitation, natural frequencies, mode shapes, mode summation, resonance and damping effects. Comparisons of analytical analysis with FE visual results (animation, for example) reinforce students' understanding of vibration theory learnt in class.

Key words: vibration, finite element

1 Introduction

In most mechanical systems and structures, vibration is harmful and should be avoided. Unbalance-induced blade and disk vibrations in turbines can cause spectacular mechanical failures. Passengers endure the annoying oscillation as a car rides over a bumpy road. Vehicle structure-borne cabin noise creates a nuisance to passengers. Vibration can cause more rapid wear of mechanical parts such as bearings. Machining vibrations in the cutting process can lead to a poor surface finish and reduce the life of cutting tools. Vibration in many cases is a limiting factor in machine designs [1,2]. Knowledge about vibration is desired for mechanical engineers to analyze, measure and control the harmful effects upon device performance.

An elective course on engineering vibration is recently offered for mechanical engineering seniors at the Purdue University North Central. The topic of vibration builds on previous courses in dynamics and engineering mathematics (kinematic and dynamic analysis, principles of energy, Laplace transform, eigenvalue problem, etc.). This course is developed to covers all essential fundamentals of mechanical vibration, such as modeling of single- and multiple-degree-of-freedom systems, free and forced response analysis, vibration measurement and vibration suppression. ME seniors are usually good at modeling a vibration system based upon dynamic analysis and finding the solutions of the resulting differential equations. However, it is not easy for many students to get physical insights of the equations of motion.

Various laboratory experiments have been employed in vibration course to demonstrate related topics and phenomena. Ruhala [3] describes five forced-vibration experiments developed for engineering vibration laboratory course. These experiments are built for measuring the transient or steady-state response of a lumped mass system with either single or multiple degrees of freedom. It is concluded that the laboratory experiments are effective in helping students understand the vibration theory and provide an increased level of intellectual excitement for the course. Hess [4] integrates four projects, including balancing and suppressing design, with theory in the vibration course. These projects provide students an opportunity to get experience with vibration equipments and expose students to some technical applications.

Finite Element (FE) method displays its unique abilities in simulating the performance of a mechanical part or system prior to building a physical prototype. It has been widely employed to solve problems relating to engineering vibrations. Integrating appropriate FE learning modules in teaching is an efficient way to assist students in the learning of engineering vibration. Animations and graphical plots from FEA enable students to visualize the phenomena of vibrations, enhancing their comprehension and grasp of some of critical concepts. Jenkins [5] develops a series of computer simulation modules for vibration course, including a topic on simulating an earthquake on a 4-DOF structure with actual acceleration data from 1940 E1 Centro earthquake. These modules can improve the students' comprehension of some of the complex topics, such as the interaction of multiple degrees o freedom.

This paper presents an effort to make use of FE models to provide a means for students to visualize some critical concepts in the engineering vibration course. The developed modules demonstrate the characteristics of free and forced vibration, natural frequencies and mode shapes, mode summation, resonance and damping effects. Student feedback to integrate FE visual results with vibration theory is positive.

2. FE learning modules

There are two kinds of vibrations: one is free and the other is forced vibration. Free vibration takes place under the action of the internal restoring forces and there is no external force present in this process. A system in free vibration oscillates at its natural frequency, which is a property of the system and is determined by its stiffness and mass ($\varpi_n = \sqrt{k/m}$). Forced vibration occurs when a system is subjected to an external excitation, which could be either periodic or nonperiodic force.

It is well known that both kinds of vibration can be modeled mathematically using second-order ordinary differential equations with constant coefficients. Many students are reluctant to solve differential equations for symbolic underlying relationships but prefer to plug in numbers into equations as early as possible. By solving equations this way, students miss the observations of the system behavior from the engineering point of view. It is beneficial for students to understand the effects of changes in the system properties (mass, stiffness and damping coefficient) on the system outputs (the amplitude of vibration, for example).

A simple spring-mass-damper FE model, as shown in Figure 1(a), is built using ANSYS to demonstrate some critical concepts in vibrations. It is convenient to modify the FE model to simulate different vibration behavior such as undamped and damped free vibration, undamped and damped forced vibration, and response to sinusoidal base excitation or to impulse excitation.

2.1. Free vibration

Figure 1(b) plots the free response of the system without damping involved (c = 0) to the impulse excitation shown in Figure 1(a). It is obvious that the vibration amplitude remains unchanged with time, which is one characteristic of undamped free vibration. We then introduce damping to the system to investigate the damping effect on vibration responses. Figure 1(c) shows the free response to the same impulse excitation after an addition of 12.5% viscous damping to the system. The vibration amplitude, instead of remaining as a constant, decreases exponentially with time because of the factor $e^{-\zeta\omega_n t}$ in the analytical response. The damping ratio is less than 1 in this case and the corresponding motion is referred to as underdamped motion. Underdamped vibration is exhibited in many mechanical systems and constitutes the most common case. Students comprehend the damping effect by comparing the frequency of damped vibration ω_d in Figure 1(c) with the undamped natural frequency ω_n in Figure 1(b). Students also learn to evaluate the damping ratio from the values measured off the plot at two successive peaks in Figure 1(c). Figure 1(d) shows the free response when the damping ratio is further increased to 1.25. The damping ratio is greater than 1 in this case, leading to an overdamped motion. It is clear that the overdamped motion does not involve any oscillation.

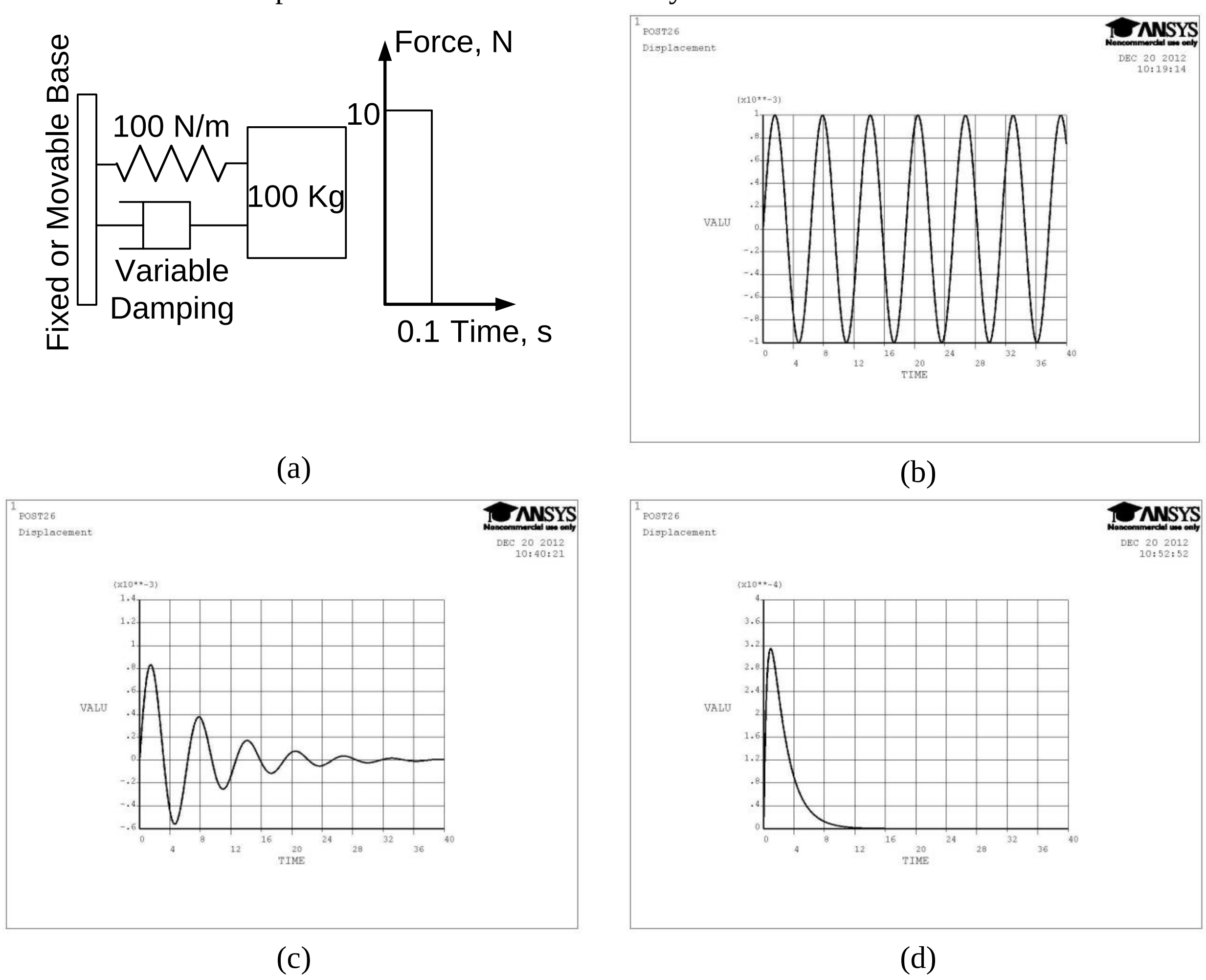
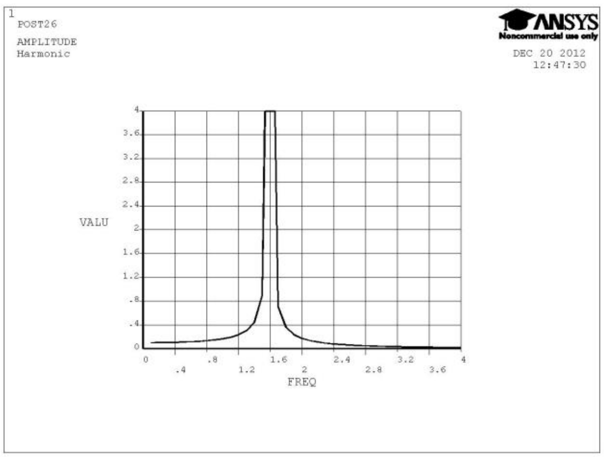


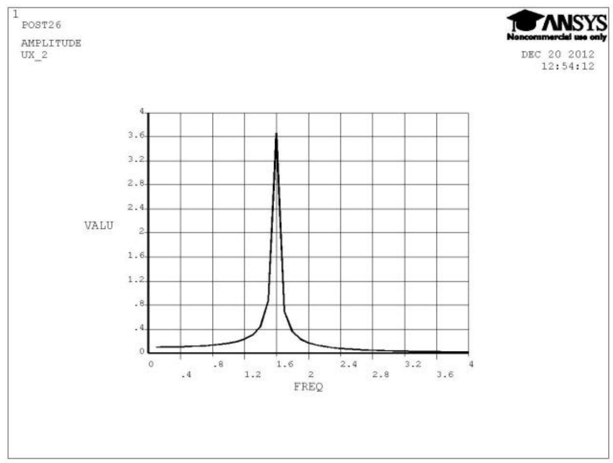
Figure 1 (a) Single degree of freedom system and impulse input, (b) undamped free response, ζ = 0, (c) underdamped free response, ζ = 0.125, and (d) overdamped free response, ζ = 1.25.

2.2. Forced vibration

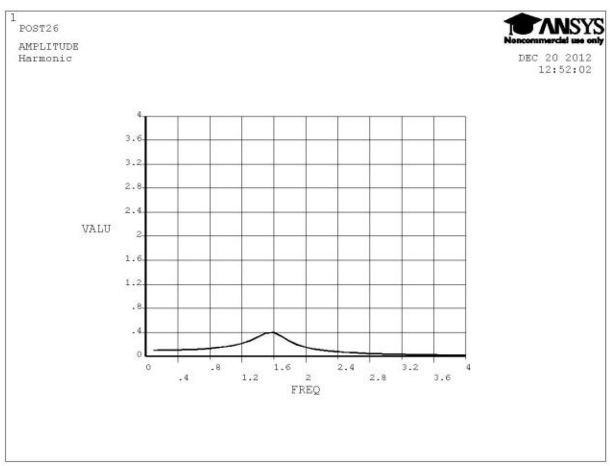
Resonance is a critical concept related to forced vibration and it occurs when a periodic external force is applied to a system having natural frequency equal to the driving frequency. Damping is of great importance in limiting the amplitude of oscillation at resonance. Students learn to understand forced vibration by applying external force to the above-mentioned FE model. The frequency of the external force ranges from 0 to 100 Hz. Figure 2 shows the forced responses for three different damping levels, i.e., 0, 12.5% and 125%, respectively. It is important from design point of view to note how the amplitude of the forced response is affected by the damping ratio. As the damping ratio decreases, the peak value increases and becomes shapers, see Figures 2 (b) and 2(c). In the limit as ζ goes to zero, the peak climbs to an infinite value, see Figure 2(a)



(a)



(b)

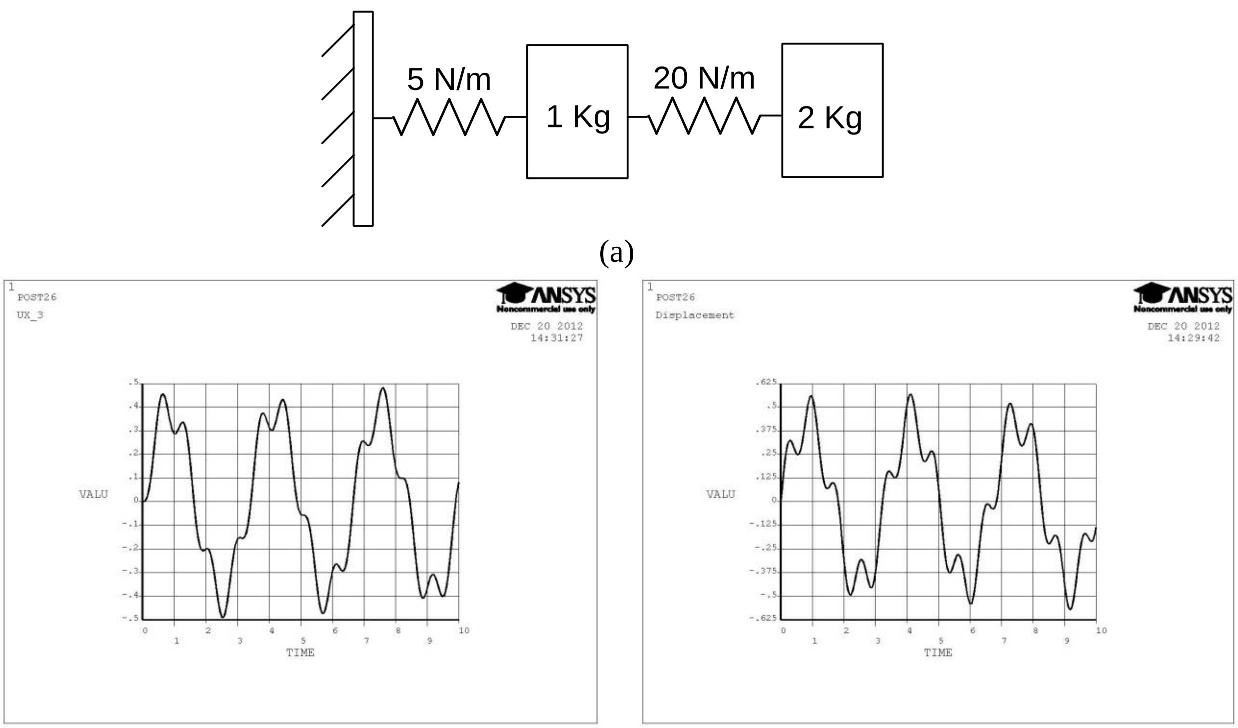


(c)

Figure 2. Forced responses for (a) $\zeta = 0$, (b) $\zeta = 0.125$, and (c) $\zeta = 1.25$. The scales used in the three plots are the same for comparison purpose.

2.3. Two degrees of freedom system
In moving from single degree of freedom systems to two and more degrees of freedom systems, two important physical phenomena result. More than one degree of freedom means more than one natural frequency, and for each of the natural frequencies, there corresponds a natural state of vibration with a displacement configuration known as the mode shape. Coupling due to spring forces and accelerations is also an important concept in the multiple degrees of freedom system (MDOF) analysis.

Although the mathematical concepts of eigenvalues and eigenvectors of computational matrix theory are readily accompanied by engineering students, interactions of multiple degrees of freedom are difficult for many students to grasp. The FE model of the system with two degrees of freedom is built as shown in Figure 3(a). Modal analysis is performed to determine the two natural frequencies and mode shapes: f_1 = 0.20255Hz, f_2 = 0.88443Hz, $\mathbf{u}_1 = [0.56040\ \ 0.60984]^T$, and $\mathbf{u}_2 = [-0.43122\ \ 0.79252]^T$, which are consistent with those from analytical analysis. The computed mode shapes are animated in ANSYS post-processing to assist students to visualize the relative motions between the two masses prior to introducing the mode summation method. The same impulse excitation as shown in Figure 1(a) is applied to the mass of 2 Kg in Figure 3(a) to study the coupling effect on the free responses. Figures 3(b) and (c) show the motions of the masses of 1 Kg and 2 Kg respectively. Students are encouraged to take a further step to perform vibration analysis for a multiple degrees of freedom system (horizontal vibration of a four-story building, for example).



(a)

(b) (c)

Figure 3 (a) Two degrees of freedom system, (b) free response of the mass of 1Kg, and (c) free response of the mass of 2Kg.

3. Conclusions

This paper presents an effort to demonstrate critical concepts in vibration course using FE modules. Students benefit from FE visual results (i.e. free and forced responses for different damping levels, animation of mode shapes, etc.) in grasping physical insights of engineering vibration. The positive effect of integrating FE modules into vibration course is confirmed by students. Torsional vibration FE modules will be developed in future offers of the vibration course.

Acknowledgements

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